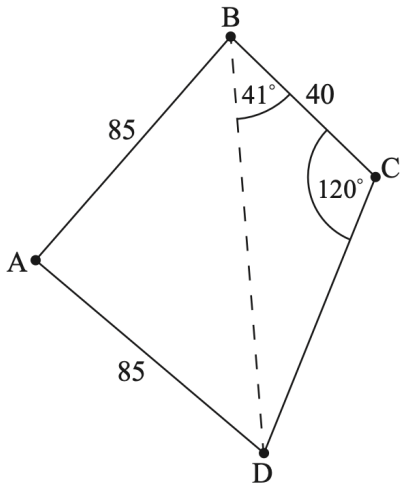
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Q1.[Maximum marks: 17]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$\mathrm{AB} = 85\,\mathrm{m},\ \mathrm{AD} = 85\,\mathrm{m},\ \mathrm{BC} = 40\,\mathrm{m},\ \mathrm{C\hat{B}D} = 41^\circ,\ \mathrm{B\hat{C}D} = 120^\circ$$

diagram not to scale



(a) (i) Write down the value of angle BDC.

(ii) Hence use triangle BDC to find the length of path BD. [4]

(b) Calculate the size of angle $\mathrm{B\hat{A}D}$, correct to five significant figures. [3]

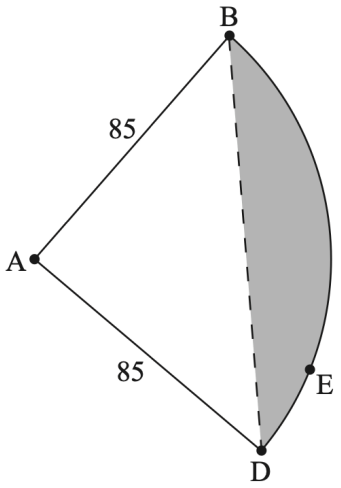
The size of angle $\mathrm{B\hat{A}D}$ rounds to 77°, correct to the nearest degree. Use $\mathrm{B\hat{A}D} = 77^\circ$ for the rest of this question.

(c) Find the area bounded by the path BD, and fences AB and AD. [3]

(This question continues on the following page)

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



(d) Write down the distance from A to E. [1]

(e) Find the perimeter of the proposed park, ABED. [3]

(f) Find the area of the shaded region in the proposed park. [3]

Q2. [Maximum marks: 16]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12300
2	12669

(a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

(b) (i) Write down the common ratio of the sequence.

(ii) Find an expression for u_n.

(iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [4]

In the first year there were 10380 places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

(c) Write down an expression for v_n. [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

(d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

(e) Find k. [3]

(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

Q3. [Maximum marks: 15]

A cafe makes x litres of coffee each morning. The cafe's profit each morning, C, measured in dollars, is modelled by the following equation

$$C = \frac{x}{10}\left(k^2 - \frac{3}{100}x^2\right)$$

where k is a positive constant.

(a) Find an expression for $\frac{dC}{dx}$ in terms of k and x. [3]

(b) Hence find the maximum value of C in terms of k. Give your answer in the form pk^3, where p is a constant. [4]

The cafe's manager knows that the cafe makes a profit of $\$426$ when 20 litres of coffee are made in a morning.

(c) (i) Find the value of k.

(ii) Use the model to find how much coffee the cafe should make each morning to maximize its profit. [3]

(d) Sketch the graph of C against x, labelling the maximum point and the x-intercepts with their coordinates. [3]

The manager of the cafe wishes to serve as many customers as possible.

(e) Determine the maximum amount of coffee the cafe can make that will not result in a loss of money for the morning. [2]

Q4. [Maximum marks: 19]

A medical centre is testing patients for a certain disease. This disease occurs in 5% of the population.

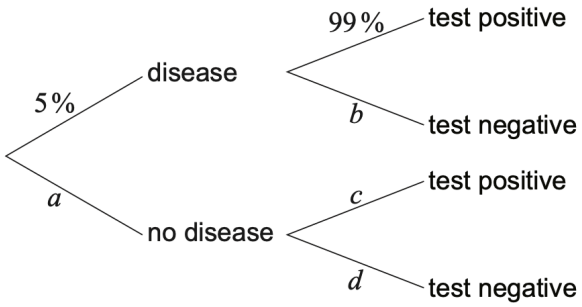
They test every patient who comes to the centre on a particular day.

(a) State the sampling method being used. [1]

It is intended that if a patient has the disease, they test "positive", and if a patient does not have the disease, they test "negative".

However, the tests are not perfect, and only 99% of people who have the disease test positive. Also, 2% of people who **do not** have the disease test positive.

The tree diagram shows some of this information.



(b) Write down the value of

(i) a.

(ii) b.

(iii) c.

(iv) d. [4]

(This question continues on the following page)

(c) Use the tree diagram to find the probability that a patient selected at random

(i) will not have the disease and will test positive.

(ii) will test negative.

(iii) has the disease given that they tested negative. [8]

(d) The medical centre finds the actual number of positive results in their sample is different than predicted by the tree diagram. Explain why this might be the case. [1]

The staff at the medical centre looked at the care received by all visiting patients on a randomly chosen day. All the patients received at least one of these services: they had medical tests (M), were seen by a nurse (N), or were seen by a doctor (D). It was found that:

- 78 had medical tests,
- 45 were seen by a nurse;
- 30 were seen by a doctor;
- 9 had medical tests and were seen by a doctor and a nurse;
- 18 had medical tests and were seen by a doctor but were not seen by a nurse;
- 11 patients were seen by a nurse and had medical tests but were not seen by a doctor;
- 2 patients were seen by a doctor without being seen by nurse and without having medical tests.

(e) Draw a Venn diagram to illustrate this information, placing all relevant information on the diagram. [3]

(f) Find the total number of patients who visited the centre during this day. [2]

Q5. [Maximum marks: 13]

The mean depth, D, in metres, of a mountain lake fluctuates in a yearly cycle and can be modelled by the function

$$D(t) = a\cos(kt) + b,$$

where t is the elapsed time, in months, since the beginning of an autumn season.

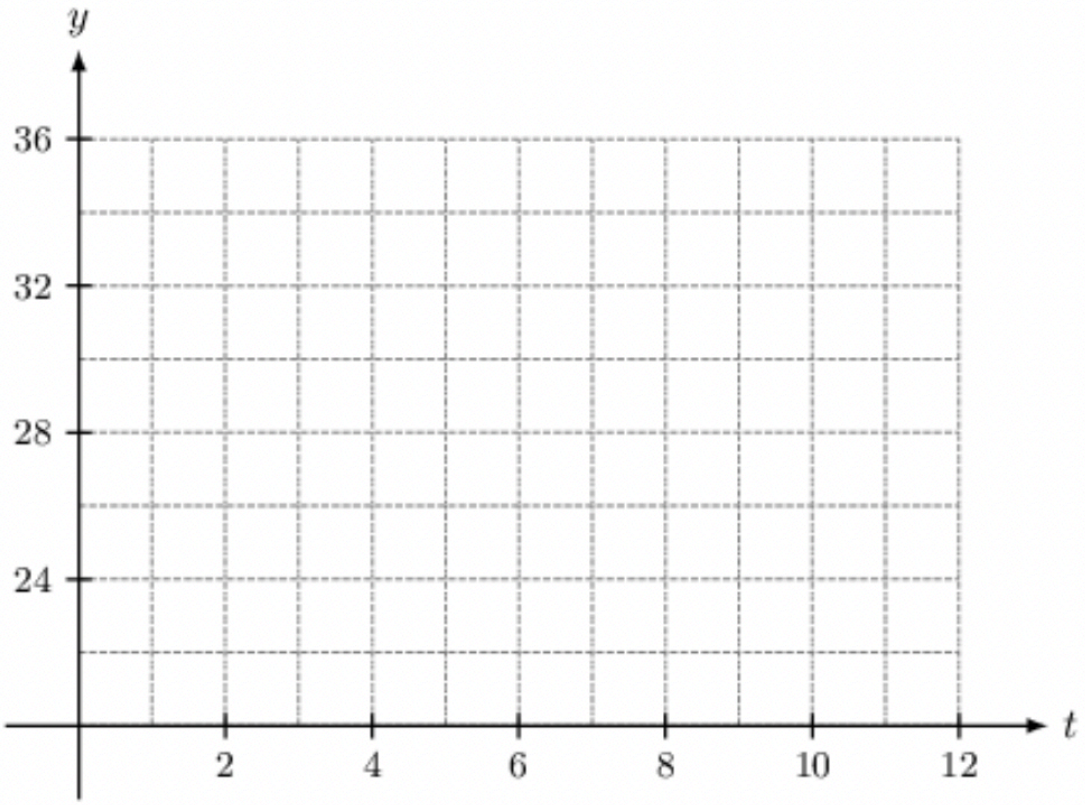
The mean depth of the lake on month 1 is 33.2 m and on month 5 is 22.8 m.

(a) Find the value of k, in degrees. [2]

(b) Set up a pair of equations and find the value of a and the value of b. Give your answers correct to **the nearest integer**. [3]

(c) Hence find the mean depth of the lake on month 8. [2]

(d) Draw the graph of $y = D(t)$ on the grid below, for one full year, indicating clearly the minimum and maximum points. [4]



(e) Determine the total amount of time in one year that the mean depth is expected to be lower than 26 metres. [2]