

## AIHL\_Paper\_3\_QP [55 marks]

1. [Maximum mark: 24]

(a) [3]

Markscheme

$$a = 70.35 \text{ OR } 70.4 \quad A1$$

$$b = 73.45 \text{ OR } 73.5 \quad A1$$

$$c = 5.45 \quad A1$$

**Note:** Follow through within the question from their (labelled) value of  $b$ , when finding  $c$  (applies both if  $b$  is incorrect or correct to a different level of accuracy).

*[3 marks]*

(b) [1]

Markscheme

**EITHER**

median/average dart score has increased *R1*

**OR**

Darts players are showing (some) improvement *R1*

**OR**

Spread in average dart score has decreased *R1*

**OR**

Players have become more similar *R1*

**Note:** Answers without context “e.g. the median is higher in year 2” score *R0*.

*[1 mark]*

(c) [1]

Markscheme

**EITHER**

(difference in averages is approximately) normally distributed *R1*

**OR**

the individuals’ changes in mean are independent *R1*

**Note:** Do not award *R1* if there is also a reference to variances being pooled / equal variances or to independence between the two years or anything else which shows a misunderstanding of the test used.

*[1 mark]*

(d.i) [2]

Markscheme

let  $\mu_d$  represent the mean of the differences from table 1

(e.g. column 2-column 1)

$H_0 : \mu_d = 0$  *A1*

$$H_1 : \mu_d > 0 \quad A1$$

**Note:** Condone an alternative hypothesis of  $H_1 : \mu_d < 0$ , provided it is consistent with their subsequent work (e.g. their “differences” are year 1 – year 2).

Accept hypotheses communicated in words, e.g. “ $H_0$  : there has been no change in the mean of the players’ averages”. The word “mean” must be seen in any word-based response.

Allow  $\mu_1 = \mu_2$  and  $\mu_2 > \mu_1$ .

Allow a compound null hypothesis (i.e.  $\mu_d \leq 0$  or equivalent).

Condone  $\mu = 0$  or  $\mu_0 = 0$  for null hypothesis and equivalent for alternative hypothesis.

**[2 marks]**

(d.ii) [2]

Markscheme

finding/entering differences in ‘means’ across both years (M1)

$$p\text{-value} = 0.134 (0.133797\dots) \quad A1$$

**[2 marks]**

(d.iii) [2]

Markscheme

$$0.134 > 0.1, \quad R1$$

(so do not reject  $H_0$ )

there is insufficient evidence to suggest that the darts players' averages have increased **A1**

**Note:** Do not award **ROA1**. Award **A0** if the conclusion is not in context or if the null hypothesis was logically incorrect (i.e. an error of confusing null and alternative rather than just an error in communication).

**[2 marks]**

(e)

(e.i) [2]

Markscheme

decrease the significance level **A2**

**Note:** Award **A1** if type I error has been defined (e.g. As rejecting  $H_0$  when  $H_0$  is true) but correct answer not seen.

Increased sample size is incorrect.

If more than one answer is given, mark only the first one.

**[2 marks]**

(e.ii) [2]

Markscheme

increase the sample size / increase the significance level **A2**

**Note:** Condone "Collect more data".

If more than one answer is given, mark only the first one.

**[2 marks]**

(f)

(f.i) [3]

Markscheme

<b>Year 1 mean</b>	<b>Year 2 mean</b>	<b>Difference</b>	<b><i>P</i></b>	<b><i>N</i></b>
68.1	70.1	2.0	<b>6</b>	
72.2	72.0	-0.2		1
65.8	65.0	-0.8		2
73.7	74.9	1.2	3	
69.8	68.0	-1.8		<b>5</b>
64.5	68.0	3.5	<b>7</b>	
69.2	70.6	1.4	<b>4</b>	
64.9	92.9	28	8	
<b>Total</b>			28	<b>8</b>

$$D = 4 \quad A1$$

$$B = 5 \quad A1$$

$$A = 6, C = 7 \quad A1$$

[3 marks]

(f.ii)

[1]

Markscheme

Year 1 mean	Year 2 mean	Difference	<i>P</i>	<i>N</i>
68.1	70.1	2.0	<b>6</b>	
72.2	72.0	-0.2		1
65.8	65.0	-0.8		2
73.7	74.9	1.2	3	
69.8	68.0	-1.8		<b>5</b>
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64.9	92.9	28	8	
<b>Total</b>			28	<b>8</b>

$$(\Sigma N) = 8 \quad A1$$

[1 mark]

(g.i)

[1]

Markscheme

( $H_1$  : ) the (population's) median (three dart average) increased (from year 1 to year 2). **A1**

**[1 mark]**

(g.ii) [1]

Markscheme

( $T =$ ) 8 **A1**

**[1 mark]**

(g.iii) [2]

Markscheme

( $T =$ ) 8 > 5 **R1**

**Note:** Condone  $8 \geq 5$ .

(so do not reject  $H_0$  (allow 'accept  $H_0$ '))

there is insufficient evidence to suggest that the median of the players' averages have increased **A1**

**Note:** Do not award **ROA1**. Award **A0** if the conclusion is not in context.

Nearly anything better than "Do not reject  $H_0$ " will be accepted as being in context.

*[2 marks]*

(h) [1]

Markscheme

**EITHER**

she could repeat the test(s) the following year **R1**

**OR**

she could repeat the test(s) with a different sample (for the same years)  
**R1**

**OR**

apply test-retest **R1**

**Note:** Increasing sample size is NOT a test for reliability. Any reference to this would score **R0**.

*[1 mark]*

2. [Maximum mark: 31]

(a)

(a.i) [2]

Markscheme

attempt to substitute  $x = 5$ ,  $y = 5$  into either of the given equations  
**(M1)**



[1 mark]

(b.ii) [3]

Markscheme

attempt to put  $\frac{dx}{dt}$  (or  $\frac{dy}{dt}$ ) equal to zero and solve (M1)

$$\frac{dx}{dt} = 0 \Rightarrow x\left(2 - \frac{y}{2}\right) = 0, \quad \frac{dy}{dt} = 0 \Rightarrow y(-3 + x) = 0$$

$$p = 3 \text{ (accept } x = 3) \quad A1$$

$$q = 4 \text{ (accept } y = 4) \quad A1$$

[3 marks]

(c)

(c.i) [4]

Markscheme

attempt to solve by separation of variables (M1)

$$\int \frac{dx}{x} = \int 2 \, dt$$

$$\ln x = 2t + c \quad (A1)(A1)$$

**Note:** Award (A1) for each side correct.

Award (A1)(A0) if both sides are integrated correctly but  $+c$  is not seen.

Also accept  $0.5 \ln x = t + c$  OR  $0.5 \ln 2x = t + c$ .

$$x = Ae^{2t} \text{ (accept } x = e^{2t+c_A}) \quad A1$$

**[4 marks]**

(c.ii) [1]

Markscheme

$$y = Be^{-3t} \text{ (accept } y = e^{-3t+c_B}) \quad \mathbf{A1}$$

**Note:** Accept alternative (distinct) labels for their  $A$  and  $B$ .

**[1 mark]**

(d.i) [3]

Markscheme

**METHOD 1**

attempt to find  $x^3$  and  $y^2$  in terms of  $t$

(e.g. any attempt to substitute their solutions to part (c)) **M1**

$$x^3 = A^3 e^{6t}, \quad y^2 = B^2 e^{-6t}$$

$$x^3 y^2 = A^3 e^{6t} \times B^2 e^{-6t} \quad \mathbf{A1}$$

$$= A^3 B^2 \quad \mathbf{A1}$$

$$= k \quad \mathbf{AG}$$

**Note:** There will probably be lots of students who assume what is to be proved by introducing a  $k$  earlier on. Do not penalize this.

## METHOD 2

attempting to isolate  $t$     **M1**

$$e^{2t} = \frac{x}{A} \text{ and } e^{3t} = \frac{B}{y}$$

$$e^{6t} = \frac{x^3}{A^3} \text{ OR } e^{6t} = \frac{B^2}{y^2} \quad \text{A1}$$

$$\frac{x^3}{A^3} = \frac{B^2}{y^2} \quad \text{A1}$$

$$x^3 y^2 = k \quad \text{AG}$$

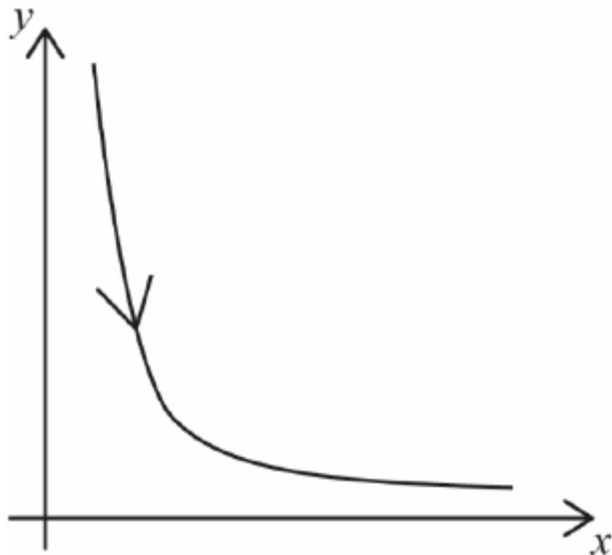
**[3 marks]**

(d.ii)

[3]

Markscheme

$$y = \sqrt{\frac{k}{x^3}} \quad \text{(A1)}$$



**A1A1**

**Note:** Award **A1** for correct shape (i.e. decreasing, convex and not touching axes), **A1** for correct direction indicated.

Ignore any trajectories not in the first quadrant.

The first **A1** can only be implied by a correct graph.

**[3 marks]**

(e) [3]

Markscheme

attempt to substitute  $x = 3 + X$ ,  $y = 4 + Y$  **OR**  
 $x = p + X$ ,  $y = q + Y$  into  $\frac{dx}{dt} = 2x - \frac{xy}{2}$  **M1**

$$X = 2(3 + X) - \frac{1}{2}(3 + X)(4 + Y)$$

$$= 6 + 2X - \frac{1}{2}(12 + 4X + 3Y + XY)$$

$$= 6 + 2X - 6 - 2X - \frac{3}{2}Y - \frac{XY}{2}$$

$$= -\frac{3Y}{2} - \frac{XY}{2} \quad \mathbf{A1}$$

The term in  $XY$  can be ignored. **R1**

$$= -\frac{3Y}{2} + \dots \approx -\frac{3Y}{2} \quad \mathbf{AG}$$

**Note:** Follow through can be awarded for the **M1** and **R1** but not the **A1**.

**[3 marks]**

(f) [1]

Markscheme

$$M = \begin{pmatrix} 0 & -\frac{3}{2} \\ 4 & 0 \end{pmatrix} \quad A1$$

[1 mark]

(g)

[4]

Markscheme

$$\text{setting } \begin{vmatrix} -\lambda & -\frac{3}{2} \\ 4 & -\lambda \end{vmatrix} = 0 \quad (M1)$$

**Note:** Setting their determinant to zero can be implied by any correct root seen.

$$\lambda^2 + 6 = 0 \quad (A1)$$

$\lambda = \pm i\sqrt{6}$  **OR**  $\pm 2.45i$  ( $\pm 2.44948\dots i$ ) **OR** noting 2 purely imaginary eigenvalues  $A1$

$\Rightarrow$  ellipse (or circle)  $R1$

**Note:** Accept "closed loops" or "oval" as descriptions of the trajectories.

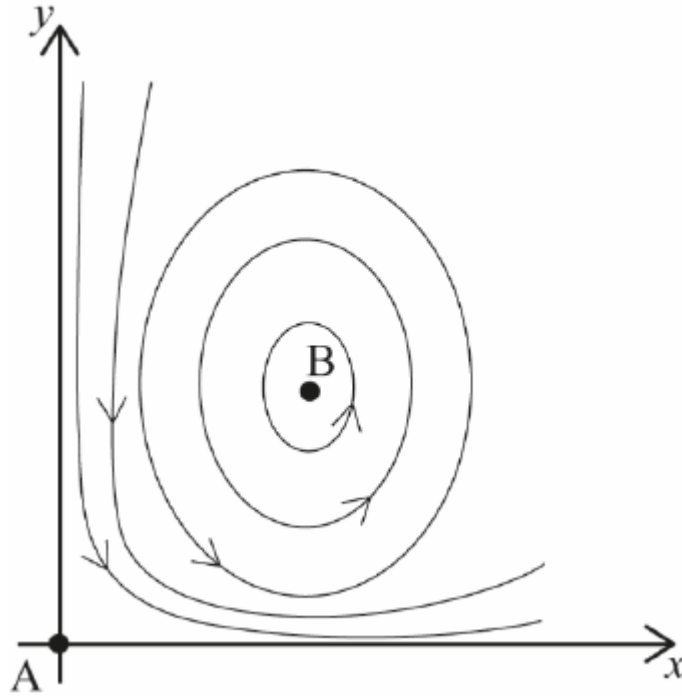
Allow follow through for the final **R1** from their eigenvalues as long as they are either both real or complex conjugates. If there are two real eigenvalues the description should be a "node" and if there are two complex eigenvalues the description should be a "focus" or a "spiral".

[4 marks]

(h)

[3]

Markscheme



**A1A1A1**

**Note:** Award **A1**: At least one ellipse, oval or closed loop seen. Allow parts of ellipses shown (as in the diagram) as long as at least one complete loop observed.

Award **A1**: Arrows showing correct direction seen on at least one loop. Must be anti-clockwise on a closed loop.

Award **A1**: Ellipses remaining entirely in the 1st quadrant **AND** centre B marked in some way (either directly or via coordinates consistent with their B).

Do not follow through into part 2(h) from part 2(g).

**[3 marks]**

(i) [2]

Markscheme

The park-keeper is incorrect **A1**

The phase portrait shows that the trajectory (starting at  $(5, 5)$ ) is a closed loop (around the equilibrium point) (despite having a negative initial  $dx/dt$  in comparison with large positive  $\frac{dy}{dt}$ ) **R1**

**Note:** Do not award **A1R0**.

**[2 marks]**