AIHL_Paper_2_QP [110 marks]

1. [Maximum mark: 13]

(a) [2]

Markscheme

$\frac{40\,000}{x^2} = 400$ ***(M1)***

$x = 10$ (pesos) (since x is positive) ***A1***

[2 marks]

(b)

(b.i) [1]

Markscheme

$\left(\frac{40\,000}{50^2} =\right)\ 16$ ***A1***

[1 mark]

(b.ii) [1]

Markscheme

$(50 \times 16 =)\ 800$ (pesos) ***A1***

[1 mark]

(c.i) [2]

Markscheme

EITHER

EITHER

profit for each smoothie $= x - 20$ ***(M1)***

$P = \frac{40\,000}{x^2} \times (x - 20)$ ***A1***

OR

profit $=$ revenue $-$ costs $= nx - 20n$ ***(M1)***

$P = x \times \frac{40\,000}{x^2} - 20 \times \frac{40\,000}{x^2}$ ***A1***

Note: Do not award ***A1*** if $\frac{40\,000}{x}$ seen as first term unless explained (in part (a) or (b)), as it is given in question.

THEN

$P = \frac{40\,000}{x} - \frac{800\,000}{x^2}$ ***AG***

[2 marks]

(c.ii) [3]

Markscheme

attempt to express P ready for power rule ***(M1)***

$P = 40\,000x^{-1} - 800\,000x^{-2}$

$\frac{\mathrm{d}P}{\mathrm{d}x} = -\frac{40\,000}{x^2} + \frac{1\,600\,000}{x^3}$ **OR**
$\frac{\mathrm{d}P}{\mathrm{d}x} = -40\,000x^{-2} + 1\,600\,000\,x^{-3}$ ***A1A1***

Note: The ***(M1)*** can be awarded for either of the correct terms seen.

A1 for each correct term.

At most ***M1A1A0*** if additional terms seen.

[3 marks]

(c.iii) [2]

Markscheme

attempt to find x-value ***(M1)***

e.g. sketch of $\frac{\mathrm{d}P}{\mathrm{d}x}$ with x-intercept indicated **OR** recognition that it occurs at the maximum of P **OR** algebraic approach (requires multiplication by x^3)

$x = 40$ ***A1***

Note: $\frac{-40\,000}{x^2} + \frac{1\,600\,000}{x^3} = 0$ is insufficient to award ***M1***, this is given in the question. There must be an “attempt to find x-value”.

Award ***M1A0*** for a coordinate pair $(40,\ 500)$.

[2 marks]

(c.iv) [2]

Markscheme

attempt to substitute their x-value into equation for n ***(M1)***

$n = \frac{40\,000}{40^2}$

$= 25$ ***A1***

Note: Given the nature of the function P, the local maximum is also the global maximum. This is often the case in examinations, but should not always be assumed.

[2 marks]

2. [Maximum mark: 12]

(a) [2]

Markscheme

$a = 0.9$, $b = 0.3$ and $c = 0.7$ ***A2***

Note: Award ***A1A0*** if one of the values is incorrect, ***A0A0*** otherwise.

[2 marks]

(b) [1]

Markscheme

$(0.1 \times 0.3 =) \quad 0.03$ ***A1***

[1 mark]

(c) [9]

Markscheme

P(no fail) $= 0.63$ ***(A1)***

P(one fails) $= 0.34$ ***(A1)***

P(two fail) $= 0.03$ *(A1)*

Note: The three ***A1***'s can be awarded independently

multiplying by 200 *(M1)*

No switch fails	One switch fails	Two switches fail
126	68	6

(A1)

degrees of freedom $= 2$ *(A1)*

Note: Award ***A1*** for df$=2$ seen anywhere and may be awarded independent of the ***M1*** mark.

The df$=2$ cannot be implied from chi squared statistic$=3.40989$

p-value $= 0.182 \quad (0.181781\ldots)$ ***A1***

$0.182 > 0.05$ ***R1***

hence insufficient evidence to reject H_0 (that the manufacturers claims are correct) ***A1***

Note: The ***R1A1*** can be awarded as follow through within part (d) from their (explicitly labelled) incorrect p-value.

An unrealistic p-value should preclude awarding the final ***R1A1***.

Accept either a conclusion to not reject the null hypothesis or the manufacturers claims are correct.

Do not award ***R0A1***.

[9 marks]

3. [Maximum mark: 13]

(a) [2]

Markscheme

C. ***A1***

Any valid reason for accepting C. or rejecting A. and B. ***R1***

for example:

- when $x = 0$ slopes have (or appear to have) zero gradient

- (slope field is) always positive for $x > 0$

Note: Allow ***A1R0***.

[2 marks]

(b) [7]

Markscheme

$\int \mathrm{e}^{2y} dy = \int\ x \mathrm{d}x$ ***(M1)***

$\frac{1}{2}\mathrm{e}^{2y} = \frac{1}{2}x^2\ (+c)$ ***(A1)(A1)***

Note: ***A1*** for left hand side, ***A1*** for right hand side.

substituting in $x = 0, y = 0$ ***(M1)***

$\frac{1}{2} = c$ ***(A1)***

Note: The substitution may be seen and credited later, however at that point the constant term may be 1.

$\mathrm{e}^{2y} = x^2 + 1$

$y = \frac{1}{2} \ln \left(x^2 + 1\right)$ ***M1A1***

Note: Award ***M1*** for use of log law.

[7 marks]

(c) [2]

Markscheme

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} \times 2x \times \frac{1}{x^2+1} \left(= \frac{x}{x^2+1}\right)$ ***M1A1***

Note: Award ***M1*** for use of chain rule, or use of implicit differentiation of the penultimate line of the answer to (b).

[2 marks]

(d) [2]

Markscheme

substitution of $\mathrm{e}^{2y} = x^2 + 1$ from part (b) into part(c)(i) or original differential equation ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{x^2+1} = \frac{x}{\mathrm{e}^{2y}}$ ***A1***

and hence $y = \frac{1}{2}\ln\left(x^2+1\right)$ is a solution for the differential equation ***AG***

Note: Only award the ***A1*** as follow-through if their $\frac{\mathrm{d}y}{\mathrm{d}x}$ is of the form $\frac{x}{x^2+c}$.

[2 marks]

4. [Maximum mark: 13]

(a)

(a.i) [2]

Markscheme

let S be the number of spaceships hit and B the number of battleships

mean$= 8.4$ ***(A1)***

$\mathrm{P}(S \leq 10) = 0.774 \quad (0.774301\ldots)$ ***A1***

[2 marks]

(a.ii) [3]

Markscheme

let S be the number of spaceships hit and B the number of battleships

attempt to add two means ***(M1)***

$4.2 + 2.3 = 6.5$

$\mathrm{P}(S + B > 10) = \mathrm{P}(S + B \geq 11)$ ***(M1)***

$0.0668 \quad (0.0668387\ldots)$ ***A1***

[3 marks]

(b)

(b.i) [1]

Markscheme

$\mathrm{E}(T) = 3 \times 4.2 + 5 \times 2.3 = 24.1$ ***A1***

[1 mark]

(b.ii) [2]

Markscheme

$\mathrm{Var}(T) = 3^2 \times 4.2 + 5^2 \times 2.3 = 95.3$ ***(M1)A1***

[2 marks]

(c) [1]

Markscheme

any valid reason ***R1***

for example:

mean is not equal to variance **OR** T cannot take all integer values

[1 mark]

(d) [4]

Markscheme

distribution of mean score is

$\mathrm{N}\left(24.1,\ \frac{95.3}{60}\right) \quad (\mathrm{N}(24.1,\ 1.58833\ldots))$ ***(A1)(A1)***

Note: Award ***A1*** for normal distribution with mean 24.1, and ***A1*** for variance $\frac{95.3}{60}$.

$\mathrm{P}\left(\overline{T} > 25\right) = 0.238 \quad (0.237576\ldots)$ ***A2***

[4 marks]

5. [Maximum mark: 18]

(a) [2]

Markscheme

$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ ***M1A1***

Note: Award ***M1*** is for a 3×3 matrix with at least one column correct.

Column order is not explicit in question and may not be labelled in candidate response; accept their correct adjacency matrix.

[2 marks]

(b) [3]

Markscheme

EITHER

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}^5$$ ***(M1)***

$$= \begin{pmatrix} 11 & 11 & 10 \\ 11 & 10 & 11 \\ 10 & 11 & 11 \end{pmatrix}$$ ***(A1)***

OR

Listing at least 8 possible walks ***(M1)***

AAAAAB, AAABAB, AAABCB, AABAAB, AABCCB, ABABAB, ABACAB, ABAAAB, ABCCCB, ABCBAB, ABCBCB ***(A1)***

THEN

11 different routes ***A1***

[3 marks]

(c.i) [1]

Markscheme

0.5^5 $\left(\frac{1}{32}, 0.03125\right)$ ***A1***

[1 mark]

(c.ii) [2]

Markscheme

EITHER

there are 11 possible walks so probability is 11×0.5^5 ***M1***

OR

total number of (equally likely) walks from A is 32, 11 end up at B ***M1***

THEN

$\frac{11}{32}$ **OR** 0.344 (0.34375) ***A1***

Note: Solutions to this part must be using the value (11) obtained from part (b) to be awarded any marks

[2 marks]

(d)

(d.i) [1]

Markscheme

$(1 \times 0.4 =)$ 0.4 ***A1***

[1 mark]

(d.ii) [1]

Markscheme

$(0.5 \times 0.5 =)\quad 0.25$ ***A1***

[1 mark]

(d.iii) [1]

Markscheme

$(0.5 \times 0.5 + 0.5 \times 0.5 =)\quad 0.5$ ***A1***

[1 mark]

(e) [4]

Markscheme

transition matrix is $\begin{pmatrix} 0 & 0.25 & 0.4 \\ 0.6 & 0.5 & 0.6 \\ 0.4 & 0.25 & 0 \end{pmatrix}$ (with order AB, AC and BC) ***(M1)(A1)***

Note: Column order is not explicit in question and may not be labelled in candidate response; accept their correct transition matrix.

Accept the transposed matrix.

$\begin{pmatrix} 0 & 0.25 & 0.4 \\ 0.6 & 0.5 & 0.6 \\ 0.4 & 0.25 & 0 \end{pmatrix}^5$ ***(M1)***

$$= \begin{pmatrix} 0.22215 & 0.227275 & 0.23239 \\ 0.54546 & 0.54545 & 0.54546 \\ 0.232399 & 0.227275 & 0.22215 \end{pmatrix}$$

$0.232 \quad (0.23239)$ ***A1***

[4 marks]

(f) [3]

Markscheme

(Taking a high power of a matrix)

long term probabilities are $0.227275, 0.545455$ and 0.227275 ***(M1)***

B and 0.545 (54.5% $\frac{6}{11}$) ***A1A1***

Note: Award ***(M0)A0A0*** for an unsupported answer of "B" (with either no probability or an incorrect probability).

[3 marks]

6. [Maximum mark: 15]

(a) [5]

Markscheme

heights, $0, \ 4, \ 1.75, \ 3$ and 3.75 seen ***(A2)***

Note: Award ***A1A0*** if **two** of $1.75, 3$ or 3.75 are seen.

attempt to use trapezoidal rule formula for their heights ***(M1)***

$\frac{1}{2} \times 1 \times \{0 + 4 + 2(1.75 + 3 + 3.75)\}$ ***(A1)***

Note: Award ***(M1)(A1)*** for correctly expressing this as 3 trapezoids and a triangle. The "$\times 1$" need not be seen.

$= 10.5\,(\mathrm{m}^2)$ ***A1***

[5 marks]

(b) [3]

Markscheme

$-\frac{1}{12}x^3 + x^2 + c$ ***A1A1A1***

[3 marks]

(c) [4]

Markscheme

$\int_0^4 \left(-\frac{1}{4}x^2 + 2x\right) \mathrm{d}x + 1 \times 4 + \frac{1}{2} \times 7 \times 4$ ***(A1)(M1)(A1)***

Note: Award ***A1*** for correct area of rectangle **OR** triangle, ***M1*** for substituting correct limits into given integral (may be seen in part (b)), and ***A1*** for entire expression correct.

$= 10.6666\ldots \quad + 4 + 14$

$= 28\frac{2}{3}\,(\mathrm{m}^2) \quad \left(\frac{86}{3}\right)$ ***A1***

Note: The answer must be **exact** for the ***A1*** to be awarded. For an answer of 28.7 or 28.66 award ***(A1)(M1)(A1)A0***.

[4 marks]

(d) [3]

Markscheme

(Total area using part (a) =) 28.5 ***(A1)***

Percentage error $\left|\frac{28.5-28.6666\ldots}{28.6666\ldots}\right| \times 100$ ***(M1)***

Note: if their trapezoid value is incorrect but is used correctly in the percentage error formula, award at most ***A0M1A0***. If it is clear from the answer that $\times 100$ has been used, then condone the omission and award the ***M*** mark.

$= 0.581\,(\%) \quad (0.581395\ldots)$ ***A1***

(accept 0.697 from use of 28.7)

[3 marks]

7. [Maximum mark: 14]

(a.i) [1]

Markscheme

15 ***A1***

[1 mark]

(a.ii) [3]

Markscheme

EITHER

attempt to use arithmetic series formula ***(M1)***

OR

attempt to set up simultaneous equations ***(M1)***

OR

attempt to use quadratic regression ***(M1)***

$(T_k =)\frac{1}{2}k^2 + \frac{1}{2}k$ ***A1A1***

Note: Condone variable change (eg in quadratic regression).

Accept $a = \frac{1}{2},\ b = \frac{1}{2}$.

[3 marks]

(b.i) [1]

Markscheme

$(15 + 10 =)\ 25$ ***A1***

[1 mark]

(b.ii) [2]

Markscheme

$\frac{k(k+1)}{2} + \frac{(k-1)((k-1)+1)}{2}$ **OR**

$\frac{1}{2}k^2 + \frac{1}{2}k + \frac{1}{2}(k-1)^2 + \frac{1}{2}(k-1)$ ***(A1)***

$= k^2$ ***A1***

[2 marks]

(c) [3]

Markscheme

one correct product of probabilities seen: $\frac{15}{25} \times \frac{10}{24}$ **OR** $\frac{10}{25} \times \frac{15}{24}$ ***(A1)***

adding their products ***(M1)***

$\frac{15}{25} \times \frac{10}{24} + \frac{10}{25} \times \frac{15}{24}$

$= \frac{1}{2}$ ***A1***

[3 marks]

(d) [4]

Markscheme

attempt to add two products of probabilities involving k only ***M1***

(these may be incorrect or in terms of T_k)

$\frac{\frac{k}{2}(k+1)}{k^2} \times \frac{\frac{k}{2}(k-1)}{k^2-1} + \frac{\frac{k}{2}(k-1)}{k^2} \times \frac{\frac{k}{2}(k+1)}{k^2-1}$ ***A1***

further simplification consistent with given answer ***A1***

$= \frac{1}{2}$ ***A1***

hence independent of k ***AG***

[4 marks]

8. [Maximum mark: 12]

(a) [5]

Markscheme

stretch ***A1***

scale factor 3, ***A1***

y-axis invariant (condone parallel to the x-axis) ***A1***

and

stretch, scale factor 2, ***A1***

x-axis invariant (condone parallel to the y-axis) ***A1***

[5 marks]

(b) [2]

Markscheme

$\det(A) = 6$ ***A1***

$7 \times 6 = 42\,\text{cm}^2$ ***A1***

[2 marks]

(c) [3]

Markscheme

$B = AX$ ***(A1)***

$X = A^{-1}B$ ***(M1)***

$$X = \begin{pmatrix} 0.866 & 0.5 \\ -0.5 & 0.866 \end{pmatrix} \left(= \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \right)$$ ***A1***

[3 marks]

(d) [2]

Markscheme

Rotation ***A1***

clockwise by 30° about the origin ***A1***

[2 marks]