

AIHL_Paper_1_QP [110 marks]

1. [Maximum mark: 9]

(a) [2]

Markscheme

(1.04, 0.509) ((1.03667..., 0.509085...)) **A1A1**

[2 marks]

(b) [5]

Markscheme

attempt to make x the subject for either function **(M1)**

$x = 4y^2$, $x = \cos^{-1} y$ **A1A1**

attempt to use $V = \pi \int x^2 dy$ **(M1)**

$$V = \pi \int_0^{0.509085\dots} (4y^2)^2 dy + \pi \int_{0.509085\dots}^1 (\cos^{-1} y)^2 dy$$
 A1

[5 marks]

(c) [2]

Markscheme

= 1.15 (units³) **A2**

Note: Do not **FT** from part (b) to part (c).

Award **A1** for 1. 1. with no previous working.

[2 marks]

2. [Maximum mark: 6]

(a) [1]

Markscheme

$\frac{dy}{dx}$ is undetermined at (0, 1) **R1**

(so cannot use $y_n = y_{n-1} + h\left(\frac{x}{(x^2+1)(2y-2)}\right)$)

Note: Accept “undefined”, “indeterminate” or “division by zero” in place of “undetermined”.

[1 mark]

(b) [4]

Markscheme

$$\int (2y - 2) dy = \int \frac{x}{x^2+1} dx \quad \mathbf{M1}$$

$$y^2 - 2y = \frac{1}{2} \ln(x^2 + 1) + c \quad \mathbf{A1}$$

substituting $x = 0, y = 1$ **M1**

$$c = -1$$

$$y^2 - 2y + 1 = \frac{1}{2} \ln(x^2 + 1)$$

$$(y - 1)^2 = \frac{1}{2} \ln(x^2 + 1) \quad A1$$

$$y - 1 = \sqrt{\frac{1}{2} \ln(x^2 + 1)} \quad (\text{where positive root required as } y \geq 1)$$

$$y = 1 + \sqrt{\frac{\ln(x^2 + 1)}{2}} \quad AG$$

[4 marks]

(c) [1]

Markscheme

$$(\text{when } x = 0.1) y = 1.07 \text{ (1.07053...)} \quad A1$$

[1 mark]

3. [Maximum mark: 5]

(a) [2]

Markscheme

$$\sin(21^\circ) = \frac{17}{BF} \quad (M1)$$

$$BF = 47.4 \text{ m (47.4372...)} \quad A1$$

[2 marks]

(b) [3]

Markscheme

EITHER

$$BE = \sqrt{47.4372\dots^2 + 44^2} = 64.7015\dots \quad (A1)$$

$$\sin^{-1}\left(\frac{17}{BE}\right) \quad (M1)$$

$$= 15.2^\circ \text{ (15.2329\dots}^\circ\text{) (or 0.266 radians (0.265866\dots))}$$

A1

OR

$$AD = \sqrt{47.4372\dots^2 - 17^2} = 44.2865\dots \quad (A1)$$

$$DB = \sqrt{64.7015\dots^2 + 44^2} = 62.4832\dots$$

$$\tan^{-1}\left(\frac{17}{62.4832\dots}\right) \quad (M1)$$

$$= 15.2^\circ \text{ (15.2329\dots}^\circ\text{) (or 0.266 radians (0.265866\dots))}$$

A1

[3 marks]

4. [Maximum mark: 7]

(a) [2]

Markscheme

$$\begin{array}{c}
 \text{S} \\
 \text{A} \\
 \text{B} \\
 \text{C} \\
 \text{D} \\
 \text{T}
 \end{array}
 \begin{array}{c}
 \text{S} \text{ A} \text{ B} \text{ C} \text{ D} \text{ T} \\
 \left(\begin{array}{cccccc}
 0 & 1 & 1 & 1 & \boxed{2} & 0 \\
 1 & 0 & 1 & 1 & \boxed{0} & 0 \\
 1 & 1 & 0 & 1 & 1 & 1 \\
 1 & 1 & 1 & 0 & 1 & 1 \\
 \boxed{2} & \boxed{0} & 1 & 1 & 0 & 1 \\
 0 & 0 & 1 & 1 & 1 & 0
 \end{array} \right)
 \end{array}$$

$$SD = DS = 2 \quad A1$$

$$AD = DA = 0 \quad A1$$

[2 marks]

(b) [4]

Markscheme

attempt to calculate at least one of M^2 , M^3 and M^4 (M1)

attempt to calculate all of M^2 , M^3 and M^4 (M1)

finding at least one of the top right entries, 4, 10, 64 (A1)

78 walks A1

Note: If $SD = DS = 1$ is their answer in part (a), their **FT** answer is $(3 + 8 + 41 =) 52$ walks.

[4 marks]

(c) [1]

Markscheme

because some of the walks will pass through T , before returning to T **R1**

[1 mark]

5. [Maximum mark: 5]

(a) [1]

Markscheme

$$\frac{50 \times \pi}{180} = 0.873 \text{ (0.872664...)} \quad \mathbf{A1}$$

[1 mark]

(b) [4]

Markscheme

$$\text{volume} = 240 \left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664... \right) \quad \mathbf{M1M1M1}$$

Note: Award **M1** $240 \times \text{area}$, award **M1** for correctly substituting area sector formula, award **M1** for subtraction of the angles or their areas.

$$= 45800 \text{ (= 45811.96071)} \quad \mathbf{A1}$$

[4 marks]

6. [Maximum mark: 5]

(a) [1]

Markscheme

$$E = 5(2 \sin t)^2 (= 20 \sin^2 t) \quad A1$$

[1 mark]

(b) [2]

Markscheme

$$\frac{dE}{dt} = 40 \sin t \cos t \quad (M1)A1$$

[2 marks]

(c) [2]

Markscheme

$$t = 0.126 \quad (M1)A1$$

[2 marks]

7. [Maximum mark: 5]

(a) [2]

Markscheme

$$s_{n-1} = \sqrt{\frac{10}{9}} \times 0.0196 = 0.02066 \dots \quad (M1)A1$$

[2 marks]

(b) [2]

Markscheme

(1.463, 1.493) (M1)A1

Note: If s_n used answer is (1.464, 1.492), award **M1A0**.

[2 marks]

(c) [1]

Markscheme

95 % of the time these results would be produced by a population with mean of less than 1.5 kg, so it is likely the mean mass is less than 1.5 kg **R1**

[1 mark]

8. [Maximum mark: 6]

[6]

Markscheme

let T be the time to serve both customers and T_i the time to serve the i th customer

assuming independence of T_1 and T_2 **R1**

T is normally distributed and $T = T_1 + T_2$ **(M1)**

$E(T) = 1.5 + 1.5 = 3$ **A1**

$\text{Var}(T) = 0.4^2 + 0.4^2 = 0.32$ **M1A1**

$P(T < 4) = 0.961$ **A1**

[6 marks]

9. [Maximum mark: 7]

[7]

Markscheme

(Model A)

$$R = 3pe^{-0.5p} \quad M1$$

predicted values

p	R
1	1.8196
2	2.2073
3	2.0082

(A1)

$$SS_{res} = (1.8196 - 1.5)^2 + (2.2073 - 1.8)^2 + (2.0082 - 1.5)^2$$

(M1)

$$= 0.5263... \quad A1$$

(Model B)

$$R = 2.5pe^{-0.6p}$$

predicted values

p	R
1	1.372
2	1.506
3	1.2397

(A1)

$$SS_{res} = 0.170576... \quad A1$$

chose model B $A1$

Note: Method marks can be awarded if seen for either model A or model B. Award final **A1** if it is a correct deduction from their calculated values for A and B.

[7 marks]

10. [Maximum mark: 6]

(a) [1]

Markscheme

$$H_0 : m = 3, H_1 : m < 3 \quad A1$$

Note: Accept equivalent statements in words.

[1 mark]

(b) [2]

Markscheme

(let X be the number of fish caught)

$$P(X \leq 1 | m = 3) = 0.199 \quad M1A1$$

[2 marks]

(c) [3]

Markscheme

$$P(X \geq 2 | m = 2.5) \quad (= 1 - P(X \leq 1 | m = 2.5)) \quad M1A1$$

Note: Award **M1** for using $m = 2.5$ to evaluate a probability, award **A1** for also having $X \geq 2$.

$$= 0.713 \quad A1$$

[3 marks]

11. [Maximum mark: 5]

(a) [1]

Markscheme
3 A1
[1 mark]

(b) [1]

Markscheme
1 A1
[1 mark]

(c) [3]

Markscheme												
<table border="1"><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$f^{-1}(x)$</td><td>2</td><td>1</td><td>4</td><td>0</td><td>3</td></tr></table>	x	0	1	2	3	4	$f^{-1}(x)$	2	1	4	0	3
x	0	1	2	3	4							
$f^{-1}(x)$	2	1	4	0	3							
award A1 for 1 correct, A2 for 3 correct, A3 for all 5 correct A1A1A1												
[3 marks]												

12. [Maximum mark: 6]

(a) [2]

Markscheme
0, 1, 3 A2

Note: Award **A1** if either **1** or **3** is seen. Award **A1** for **0**, **1 000**, **3 000**

[2 marks]

(b) [4]

Markscheme

sketching a graph or using calculus (M1)

attempting to find the y coordinate at the local maximum (M1)

local max occurs at (0.451416 ..., 0.631130 ...) (A1)

$w > 0.631$ A1

[4 marks]

13. [Maximum mark: 9]

(a) [1]

Markscheme

let X be the number of worms in 1 square metre then $X \sim \text{Po}(1.2)$

$P(X = 1) = 0.361$ (0.361433 ...) A1

[1 mark]

(b) [1]

Markscheme

$P(X \geq 1) = 0.699$ (0.698805 ...) A1

[1 mark]

(c) [2]

Markscheme

if Y is the number of worms in 5 square metres then $Y \sim \text{Po}(6)$ (M1)

$$P(Y = 5) = 0.161 \quad (0.160623 \dots) \quad A1$$

[2 marks]

(d) [2]

Markscheme

EITHER

attempting to find an expression in terms of $P(X = 1)$ (M1)

$$P(X = 1)^5 = 0.00617 \quad (0.00616792 \dots) \quad A1$$

OR

let V be the number of square metres containing exactly one worm
attempting to use the binomial with $P(X = 1)$ (M1)

then $V \sim B(5, P(X = 1))$

$$P(V = 5) = 0.00617 \quad (0.00616792 \dots) \quad A1$$

Note: Accept 0.00613 from 3sf FT from part (a).

[2 marks]

(e) [3]

Markscheme

let W be the number of square metres containing at least one worm.

attempting to use binomial (M1)

then $W \sim B(5, P(X \geq 1))$ (A1)

$$P(W = 3) = 0.310 \quad (0.309572\dots) \quad A1$$

Note: Accept 0.31 for final A1. Accept 0.309 from 3sf FT from part (b).

[3 marks]

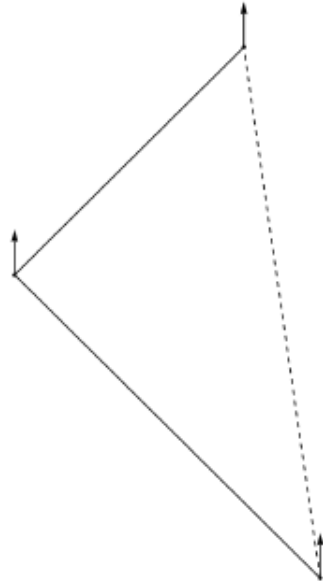
14. [Maximum mark: 5]

[5]

Markscheme

METHOD 1

diagram showing (approximately) correct directions (and order) for the 315° and 045° (A1)



recognizing right angle triangle (M1)

correct expression to find second angle in triangle (A1)

e.g. $\arctan\left(\frac{6}{8}\right)$ OR $\arctan\left(\frac{8}{6}\right)$

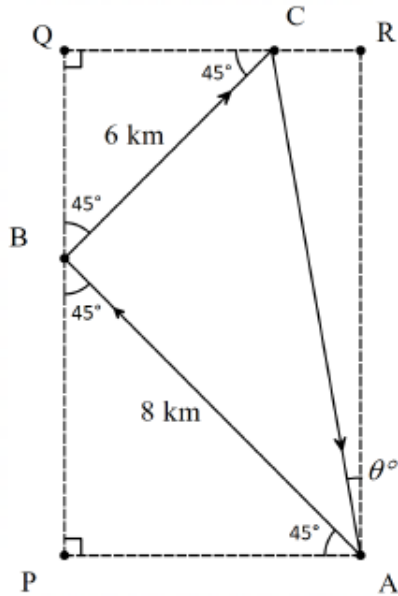
correct expression to find bearing (A1)

e.g. $\arctan\left(\frac{6}{8}\right) + 135^\circ$ OR $360^\circ - \left(\arctan\left(\frac{8}{6}\right) + 135^\circ\right)$

$= 172^\circ$ (171.869...°) A1

METHOD 2

diagram showing (approximately) correct directions (and order) for the 315° and 045° (these may be shown in reverse as the return journey) (A1)



finding the lengths marked AP, BP, CQ and BQ in the diagram (M1)

$$AP = BP = 8 \frac{\sqrt{2}}{2} = 5.6568 \dots$$

$$CQ = BQ = 6 \frac{\sqrt{2}}{2} = 4.2426 \dots$$

Note: This may be done using a vector approach.

using $\tan \theta^\circ = \frac{AP-CQ}{PB+BQ}$ or equivalent to find the direction of AC (A1)

correct expression to find bearing (A1)

$$180^\circ - \arctan \left(\frac{8 \frac{\sqrt{2}}{2} + 6 \frac{\sqrt{2}}{2}}{8 \frac{\sqrt{2}}{2} - 6 \frac{\sqrt{2}}{2}} \right)$$

$$= 172^\circ (171.869 \dots^\circ) \quad A1$$

[5 marks]

15. [Maximum mark: 7]

(a.i) [2]

Markscheme

METHOD 1

attempt to find change in height of the ball using gradient (M1)

$$\frac{a}{0.43} = (-)0.045$$

$$a = (-)0.045 \times 0.43$$

$$a = (-)0.0194(\text{m}) \quad (0.01935(\text{m})) \quad A1$$

METHOD 2

attempt to find height at back of home plate (M1)

horizontal distance to the front of the home plate = 16.6666... (m)

height at the back of the home plate = 0.045 (16.6666... + 0.43) + 2

(= 1.23065 (m))

Note: The M1 can be awarded for 16.6666... + 0.43 seen at some point.

$$(a = 1.25 - 1.23065 \dots)$$

$$(a =) (-)0.0194(\text{m}) \quad (0.01935(\text{m})) \quad A1$$

[2 marks]

(a.ii) [2]

Markscheme

$$1.25 - 0.01935 = 1.23065 \text{ (may be seen in part (a)(i))} \quad \mathbf{A1}$$

$$0.53 < 1.23065 < 1.24 \quad \mathbf{R1}$$

therefore a strike \mathbf{AG}

Note: Do not award $\mathbf{A0R1}$.

[2 marks]

(b) [3]

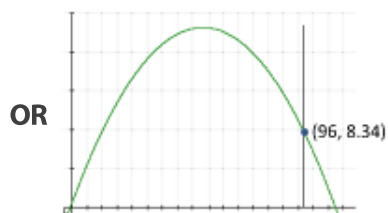
Markscheme

METHOD 1

indication of $d = 96$ in the function $h(d)$ or its graph $\mathbf{(M1)}$

EITHER

$$(h(96) =) -0.01(96)^2 + 1.04(96) + 0.66$$



THEN

$$(h(96) =) 8.34 \text{ (m)} \quad \mathbf{A1}$$

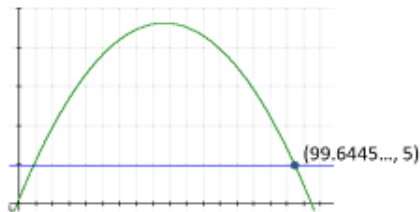
$8.34 > 5$ so the ball will go over the wall. $\mathbf{A1}$

METHOD 2

indication of $h = 5$ in the function $h(d)$ or its graph (M1)

EITHER

$$5 = -0.01d^2 + 1.04d + 0.66$$

OR**THEN**

$d = 99.6$ (m) ($99.6445 \dots$ (m)) ($d = 4.35548 \dots$ (m) may also be seen) A1

$96 < 99.6445 \dots$ so the ball will go over the wall. A1

[3 marks]

16. [Maximum mark: 5]

(a) [3]

Markscheme

$$y = \ln\left(\frac{1}{x-2}\right)$$

an attempt to isolate x (or y if switched) (M1)

$$e^y = \frac{1}{x-2}$$

$$x - 2 = e^{-y}$$

$$x = e^{-y} + 2$$

switching x and y (seen anywhere) **M1**

$$f^{-1}(x) = e^{-x} + 2 \quad \mathbf{A1}$$

[3 marks]

(b) [2]

Markscheme

sketch of $f(x)$ and $f^{-1}(x)$ **(M1)**

$$x = 2.12 \quad (2.12002 \dots) \quad \mathbf{A1}$$

[2 marks]

17. [Maximum mark: 5]

(a) [4]

Markscheme

METHOD 1

recognizing that the real part is distributive **(M1)**

$$V_T = \operatorname{Re}(2e^{3ti} + 5e^{3ti+4i})$$

$$= \operatorname{Re}(e^{3ti}(2 + 5e^{4i})) \quad \mathbf{(A1)}$$

$$\text{(from the GDC)} \quad 2 + 5e^{4i} = 3.99088 \dots e^{-1.89418 \dots i} \quad \mathbf{(A1)}$$

Note: Accept arguments differing by 2π e.g. 4. 38900 ...).

therefore

$$V_T = 3.99 \cos(3t - 1.89) \quad (3.99088 \dots \cos(3t - 1.89418 \dots))$$

A1

Note: Award the last **A1** for the correct values of A , B and C seen either in the required form or not. If method used is unclear and answer is partially incorrect, assume Method 2 and award appropriate marks eg. **(M1)A1A0A0** if only A value is correct.

METHOD 2

converting given expressions to cos form **(M1)**

$$V_T = 2 \cos 3t + 5 \cos(3t + 4)$$

(from graph) $A = 3.99$ (3.99088 ...) **A1**

$$V_T = 3.99 \cos(Bt + C)$$

either by considering transformations or inserting points

$$B = 3 \quad \mathbf{A1}$$

$$C = -1.89 \quad (-1.89418 \dots) \quad \mathbf{A1}$$

Note: Accept arguments differing by 2π e.g. 4. 38900 ...

(so,

$$V_T = 3.99 \cos(3t - 1.89) \quad (3.99088 \dots \cos(3t - 1.89418 \dots))$$

)

Note: It is possible to have $A = 3.99$, $B = -3$ with $C = 1.89$ **OR**

$A = -3.99, B = 3$ with $C = 1.25$ **OR** $A = -3.99, B = -3$ with $C = -1.25$ due to properties of the cosine curve.

[4 marks]

(b) [1]

Markscheme

maximum voltage is 3.99 (3.99088 . . .) (units) **A1**

[1 mark]

18. [Maximum mark: 7]

(a) [2]

Markscheme

use of power rule **(M1)**

$$\frac{dW}{dv} = -1.1848v^{-0.84} \text{ **OR** } -1.18v^{-0.84} \quad \text{A1}$$

[2 marks]

(b) [5]

Markscheme

$$\frac{dv}{dt} = 5 \quad \text{A1}$$

$$\frac{dW}{dt} = \frac{dv}{dt} \times \frac{dW}{dv} \quad \text{M1}$$

$$\left(\frac{dW}{dt} = -5 \times 1.1848v^{-0.84}\right)$$

when $v = 10$

$$\frac{dW}{dt} = -5 \times 1.1848 \times 10^{-0.84} \quad (M1)$$

$$-0.856 \text{ } (-0.856278\dots) \text{ } ^\circ\text{C min}^{-1} \quad A2$$

Note: Accept a negative answer communicated in words, “decreasing at a rate of...”.

Accept a final answer of $-0.852809\dots \text{ } ^\circ\text{C min}^{-1}$ from use of -1.18 .

Accept 51.4 (or 51.2) $^\circ\text{C hour}^{-1}$.

[5 marks]