

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

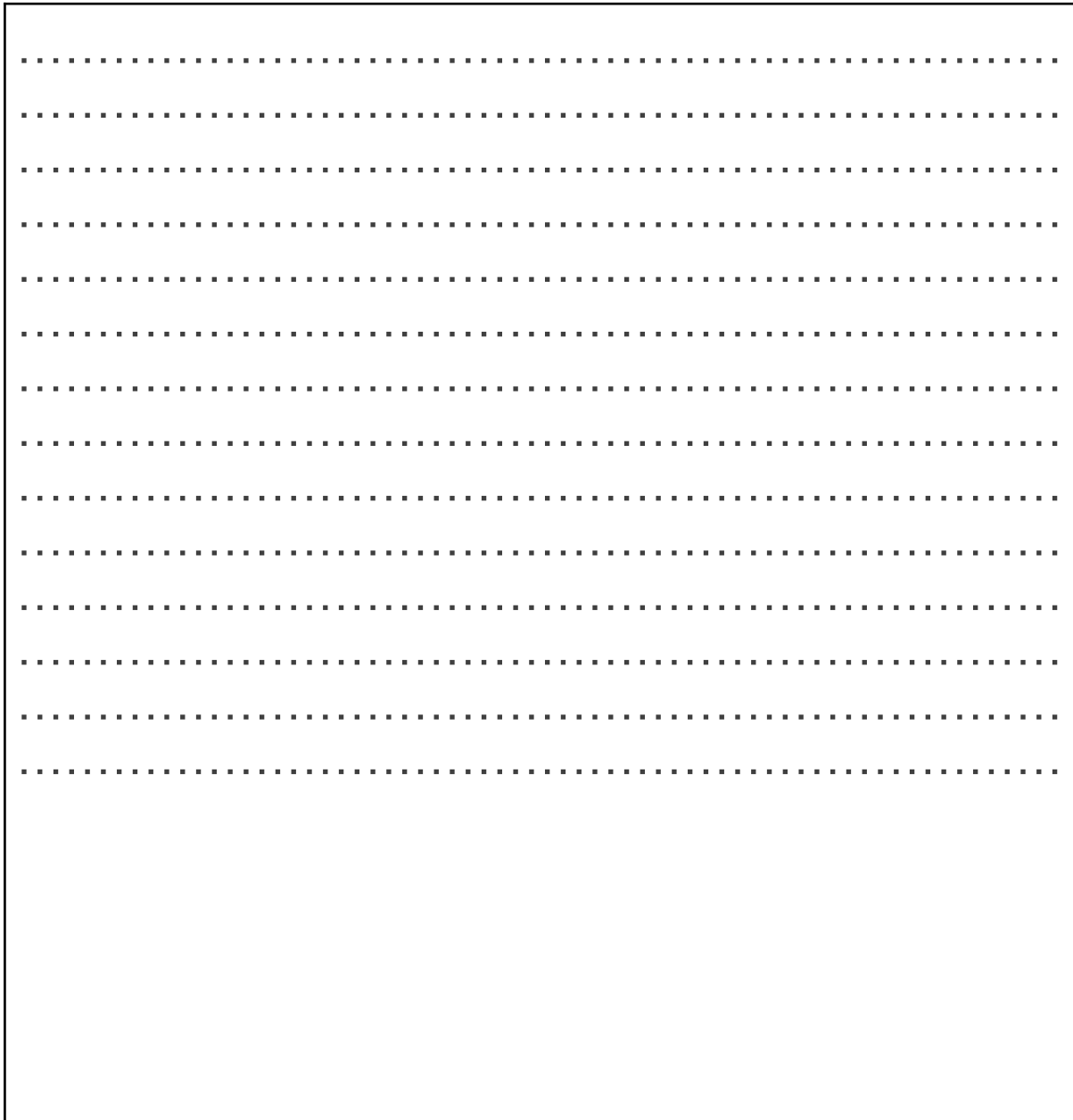
Section A

Q1 [Maximum mark: 6]

The first three terms of an arithmetic sequence are u_1 , $5u_1 - 8$ and $3u_1 + 8$

(a) Show that $u_1 = 4$. [2]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]



Q2. [Maximum mark: 7]

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$

(a) State the value of the first term, u_1 . [1]

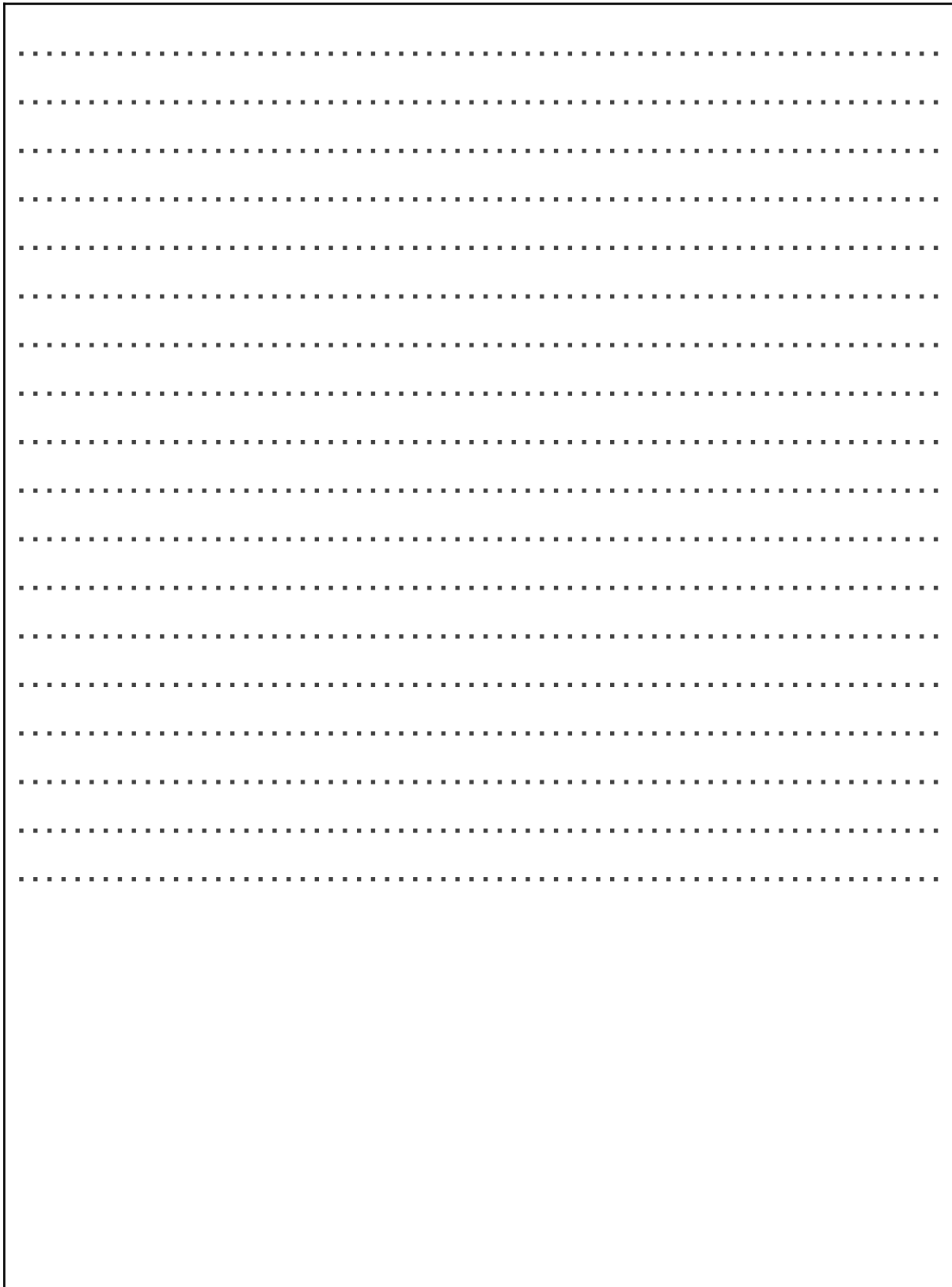
(b) Given that the n^{th} term of this sequence is -33, find the value of n . [3]

(c) Find the common difference, d . [3]

A large rectangular box containing 20 horizontal dotted lines for writing answers.

Q4. [Maximum marks 6]

Let $f'(x) = \frac{8x}{\sqrt{2x^2+1}}$. Given that $f(0) = 5$, find $f(x)$

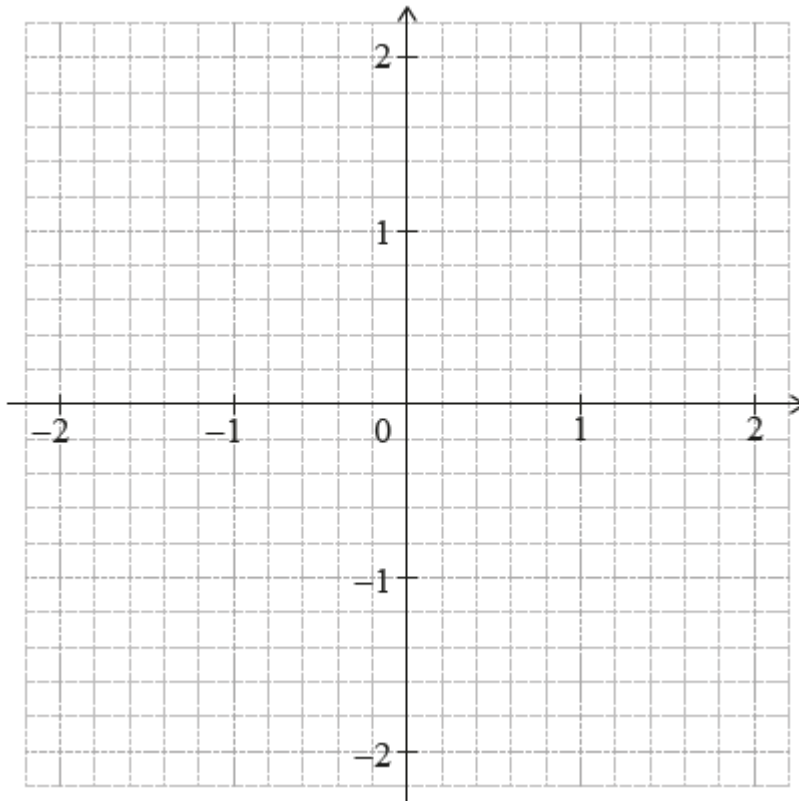


Q6. [Maximum mark: 5]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \leq x \leq 2$.

(a) Find the values of x for which $f(x) = 0$. [2]

(b) Sketch the graph of f on the following grid. [3]



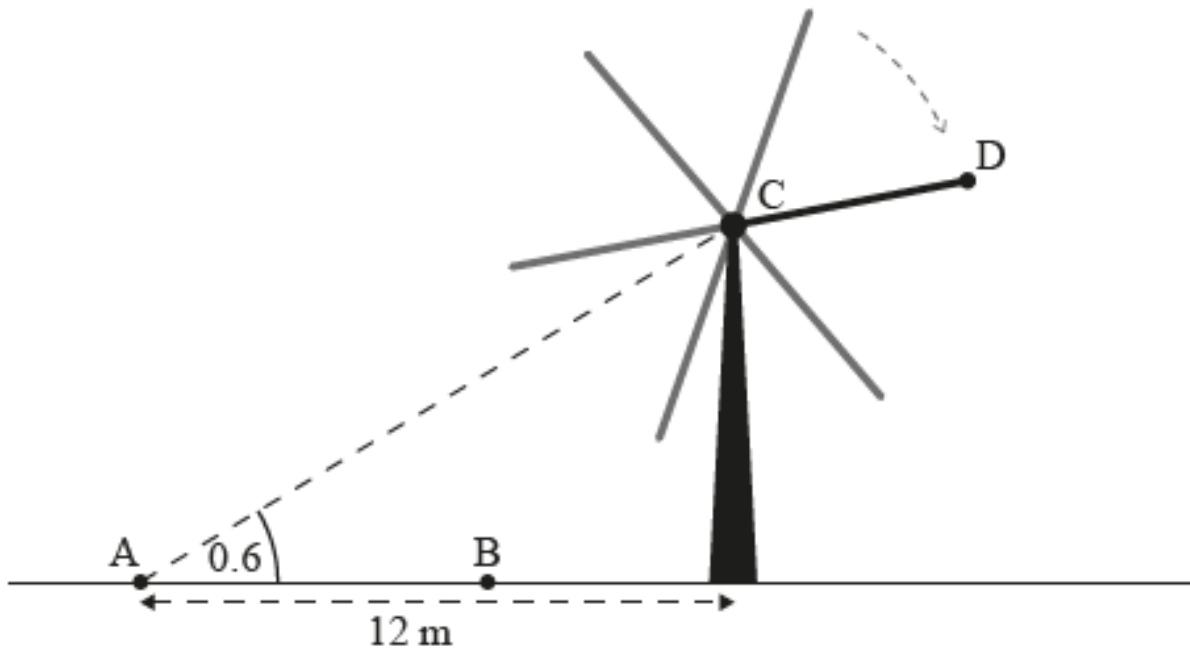
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Section B

Answer all questions in the answer booklet provided. Please start each question on a new page.

Q7. [Maximum mark: 9]

The six blades of a windmill rotate around a center point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

- (a) Given that point A is 12 meters from the base of the windmill, find the height of point C above the ground. [2]
- (b) An observer walks 7 meters from point A to point B. Find the angle of elevation of point C from point B. [2]
- (c) The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 meters, and as the blades of the windmill rotate, the end of each blade passes 2.5 meters over his head. Find the length of each blade of the windmill. [2]
- (d) One of the blades is painted a different color than the others. The end of this blade is labeled point D. The height h , in meters, of point D above the ground can be modeled by the function $h(t) = p \cos \frac{3\pi}{10}t + q$, where t is in seconds and $p, q \in \mathbb{R}$. When $t=0$, point D is at its maximum height. Find the value of p and the value of q . [3]

Q8. [Maximum mark: 18]

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$

- (a) Find $f'(x)$ [3]
- (b) The graph of f has horizontal tangents at the points where $x = a$ and $x = b$,
 $a < b$.
Find the value of a and b . [4]
- (c) Sketch the graph $y = f'(x)$. Hence explain why the graph of f has a local
maximum point at $x = a$. [3]
- (d) Find $f''(b)$. Hence explain why the graph of f has a local minimum point at $x =$
 b . [4]
- (e) The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$
intersect the point (p, q) . [4]

Q9. [Maximum mark: 15]

The temperature T degree C of water t minutes after being poured into a cup can be modelled by $T = T_0e^{-kt}$ where $t \geq 0$ and T_0, k are positive constants.

The water is initially boiling at 100°C . When $t = 10$, the temperature of water is 70°C

- (a) Show that $T_0 = 100$ [1]
- (b) Show that $k = \frac{1}{10} \ln \frac{10}{7}$. [3]
- (c) Find the temperature of the water when $t = 15$. [3]
- (d) Sketch the graph of T versus t , clearly indicating any asymptotes with their
equations and stating the coordinates of any points of intersection with the
axes. [4]
- (e) Find the time taken for the water to have a temperature of 50°C . Give your
answer correct to the nearest second. [4]