Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Q1. [Maximum mark: 7]

The function f(x) is defined for all $x \in R$. The line with equation $y = 6x - 1$ is the tangent to the graph at $x = 4$.

(a) Write down the value of f'(4) [1]

(b) Find f(4) [1]

(c) The function g is defined for all $x \in \mathbb{R}$ where $g(x)=x^2 - 3x$ and $h(x)=f(g(x))$ [3]

(d) Hence find the equation of the tangent to the graph of $h(x)$ at $x=4$. [2]

Q2.[Maximum mark: 6]

(a) Show that $(2n - 1)^2 + (2n + 1)^2 = 8n^2 + 2$, where $n \in z$.
[3]

(b) Hence, or otherwise, prove that the sum of the squares of any two consecutive odd integers is even. [3]

Q3. [Maximum mark: 7]

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

(a) Write down the value of $f'(4)$. [1]

(b) Find $f(4)$. [1]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f(g(x))$.

(c) Find $h(4)$. [2]

(d) Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

Q4. [Maximum mark: 5]

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \ldots + 1$ where $x \neq 0$ and $a, \quad b \in \mathbb{Z}^+$.

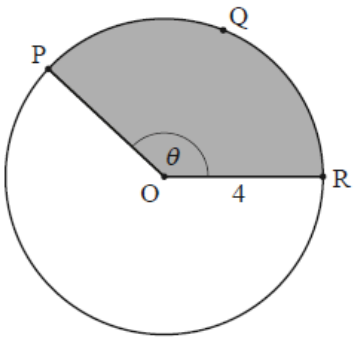
(a) Show that b = 21 [2]

(b) The third term in the expansion is the mean of the second term and the fourth term in the expansion.Find the possible values of x. [3]

Q5. [Maximum mark: 8]

The following diagram shows a circle with centre O and radius 4 cm.

diagram not to scale



The points P, Q and R lie on the circumference of the circle and $\hat{POR} = \theta$, where θ is measured in radians.

The length of arc PQR is 10 cm.

(a) Find the perimeter of the shaded sector. [3]
(b) Find θ [2]
(c) Find the area of the shaded sector. [3]

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Q6. [Maximum mark: 5]

Show that $2x - 3 - \frac{6}{x-1} = \frac{2x^2 - 5x - 3}{x-1}, \quad x \in \mathbb{R}, \quad x \neq 1.$

[5]

Section B

Answer all questions in the answer booklet provided. Please start each question on a new page

Q7.

[Maximum mark: 15]

Consider the function $f(x) = a^x$ where $x, a \in \mathbb{R}$ and $x > 0, a > 1$.

The graph of f contains the point $\left(\frac{2}{3}, 4\right)$.

(a) Show that $a = 8$. [2]

(b) Write down an expression for $f^{-1}(x)$. [1]

(c) Find the value of $f^{-1}\left(\sqrt{32}\right)$. [3]

(d) Consider the arithmetic sequence $\log_8 27$, $\log_8 p$, $\log_8 q$, $\log_8 125$, where $p > 1$ and $q > 1$.

(i) Show that $27, p, q$ and 125 are four consecutive terms in a geometric sequence.

(ii) Find the value of p and the value of q. [9]

Q8. [Maximum mark: 15]

Consider the arithmetic sequence u_1, u_2, u_3,...

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$

(a) Find the sum of the first five terms. [3]

(b) Given that $S_6 = 60$, Find u_6. [2]

(c) Hence or otherwise write an expression for u_n in terms of n. [2]

(d) Consider a geometric sequence, u_n, where $u_2 = u_1$ and $u_4 = u_6$. [3]

(e) Find the possible values of the common ratio,r. [3]

(f) Given that $u_{99} < 0$, find u_5. [2]

Q9. [Maximum mark: 12]

The function f is defined by $f(x) = \cos^2 x - 3\sin^2 x,\ 0 \le x \le \pi$.

(a) Find the roots of the equation $f(x) = 0$. [5]

(b) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]