AAHL_Paper_2_QP_(2 hrs) [110 marks]

1. [Maximum mark: 8]

(a) [3]

Markscheme

METHOD 1

attempt to use right triangle trigonometry ***(M1)***

$\tan \mathrm{B\hat{A}E} = \frac{12}{7}$ OR $\tan\left(90^\circ - \mathrm{B\hat{A}E}\right) = \frac{7}{12}$ ***(A1)***

$59.7435\ldots$

$\mathrm{B\hat{A}E} = 59.7^\circ$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

METHOD 2

attempt to find $\mathrm{B\hat{A}E}$ using sine rule OR cosine rule ***(M1)***

$\frac{\sin \mathrm{B\hat{A}E}}{12} = \frac{\sin 90}{\sqrt{12^2+7^2}}$ OR

$12^2 = 7^2 + 193 - 2 \times 7 \times \sqrt{12^2 + 7^2} \times \cos \mathrm{B\hat{A}E}$ ***(A1)***

$\mathrm{B\hat{A}E} = 59.7435\ldots$

$\mathrm{B\hat{A}E} = 59.7^\circ$ ***A1***

Note: Award ***(M1)(A1)A0*** for the equivalent radian value of 1.04.

[3 marks]

(b)

(b.i) [3]

Markscheme

METHOD 1

attempt to find DE using right angle trigonometry ***(M1)***

$\sin 59.7435\ldots^{\circ} = \frac{350}{\mathrm{DE}}$ OR equivalent ***(A1)***

$\mathrm{DE} = 405.196\ldots$

$\mathrm{CE} = 405.196\ldots + 50$

$= 455.196\ldots$

$= 455\ (\mathrm{cm})$ ***A1***

METHOD 2

Let $\mathrm{DE} = \mathrm{EF} = x$

attempt to find DE using their $\mathrm{D\hat{E}F}$ and the sine rule OR cosine rule ***(M1)***

$\frac{700}{\sin(119.487\ldots)} = \frac{\mathrm{DE}}{\sin(30.2564\ldots)}$ OR

$x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 30.2564\ldots$ ***(A1)***

$\mathrm{DE} = 405.196\ldots$

$\mathrm{CE} = 405.196\ldots + 50$

$= 455.196\ldots$

$= 455\ (\mathrm{cm})$ ***A1***

METHOD 3

Let G be the midpoint of DF

$\mathrm{EG} = \frac{7}{12} \times 350 \quad \left(= \frac{1225}{6} = 204.166\ldots\right)$ ***(A1)***

use of Pythagoras' with their EG to find DE ***(M1)***

$\mathrm{DE} = \sqrt{204.166\ldots^2 + 350^2} \quad \left(= 405.196\ldots\right)$

$\mathrm{CE} = 405.196\ldots + 50$

$= 455.196\ldots$

$= 455 \ \mathrm{(cm)}$ ***A1***

[3 marks]

(b.ii) [2]

Markscheme

$\tan\left(59.7435\ldots^\circ\right) = \frac{30}{x}$ OR $\frac{12}{7} = \frac{30}{x}$ ***(A1)***

$x = 17.5$

$\mathrm{BA} = 455.196\ldots + 17.5$

$= 472.696\ldots$

$= 473 \ \mathrm{(cm)}$ ***A1***

[2 marks]

2. [Maximum mark: 5]

[5]

Markscheme

METHOD 1

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

(discriminant $=$) $(-r)^2 - 4(4)(r-1) \quad \left(= r^2 - 16r + 16\right)$ (seen anywhere) ***(A1)***

$1.07179\ldots \quad \left(= 8 - 4\sqrt{3}\right)$ AND $14.9282\ldots \quad \left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

recognition that discriminant of $4x^2 - rx + r - 1$ is less than zero ***(M1)***

$1.07 < r < 14.9 \quad \left(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3}\right)$ ***A1***

Note: Accept $1.08 \leq r \leq 14.9$.

METHOD 2

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

EITHER

minimum when $x = \frac{r}{8} \Rightarrow \left(y =\right) 4\left(\frac{r}{8}\right)^2 - r\left(\frac{r}{8}\right) + r - 1 \; \left(> 0\right)$ ***(A1)***

attempt to solve their inequality for y (must be in terms of r and r^2) ***(M1)***

OR

$x < 1 \Rightarrow r > \frac{4x^2-1}{x-1}$ OR $x > 1 \Rightarrow r < \frac{4x^2-1}{x-1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{4x^2-1}{x-1}$ ***(M1)***

THEN

$\left(r >\right) 1.07179\ldots \quad \left(= 8 - 4\sqrt{3}\right)$ AND
$\left(r <\right) 14.9282\ldots \quad \left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

$1.07 < r < 14.9 \quad \left(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3}\right)$ ***A1***

Note: Accept $1.08 \le r \le 14.9.$

[5 marks]

3. [Maximum mark: 8]

(a) [3]

Markscheme

$\mathrm{E}(X) = \int_0^2 \frac{x}{5}\, \mathrm{d}\, x + \int_2^8 \left(-\frac{x^2}{30} + \frac{4x}{15}\right) \mathrm{d}\, x$ ***(A1)(A1)***

Note: Award ***(A1)*** $\int_0^2 \frac{x}{5} \left(\mathrm{d}\, x\right)$ and ***(A1)*** for $\int_2^8 \left(-\frac{x^2}{30} + \frac{4x}{15}\right) \left(\mathrm{d}\, x\right)$

$= \frac{2}{5} + \frac{12}{5}$

$= \frac{14}{5} \quad (= 2.8)$ ***A1***

[3 marks]

(b) [2]

Markscheme

attempt to use the expectation formula $\mathrm{E}(aX+b)=a\mathrm{E}(X)+b$ ***(M1)***

$\mathrm{E}(c-2X)=c-2\mathrm{E}(\mathrm{X}) \quad (=0)$

$c=2\mathrm{E}(X)$

$=\frac{28}{5} \quad (=5.6)$ ***A1***

[2 marks]

(c) [3]

Markscheme

recognition that median m lies between 2 and 8 e.g. using a diagram or integral ***(M1)***

$\int_0^2 \frac{1}{5}\,\mathrm{d}\,x+\int_2^m\left(-\frac{x}{30}+\frac{4}{15}\right)\mathrm{d}\,x$ OR $\int_m^8\left(-\frac{x}{30}+\frac{4}{15}\right)\mathrm{d}\,x=\frac{1}{2}$ OR $\int_2^m\left(-\frac{x}{30}+\frac{4}{15}\right)\mathrm{d}\,x=\frac{1}{10}$ ***(A1)***

$m=2.52277\ldots$

$m=2.52$ ***A1***

[3 marks]

4. [Maximum mark: 6]

(a) [3]

Markscheme

total ways $=3!\,{}^{12}C_3 \quad \left(={}^{12}P_3=1320\right)$ OR total ways together $=3!\times 10 \quad (=60)$ ***(A1)***

attempt to consider the total ways of sitting – total ways of sitting together ***(M1)***

$3!^{12}C_3 - 3! \times 10$

$= 1260$ ***A1***

[3 marks]

(b) [3]

Markscheme

METHOD 1

attempt to multiply ways of seating AVP by ways of sitting additional people ***(M1)***

AVP can sit in $3! \times 10 \quad (= 60)$ ways (may be seen in part (a))

other 3 then have $9 \times 8 \times 7 \quad \left(= {}^{9}P_3\right)$ ways to sit ***(A1)***

total ways $= 3! \times 10 \times 9 \times 8 \times 7$

$= 30240$ ***A1***

Note: Award ***(M1)(A0)A0*** for $3! \times 10 \times {}^{9}C_3 = 5040$.

METHOD 2

attempt to consider 'AVP' as one item, so 4 'items' in total ***(M1)***

${}^{10}C_4 \times 3! \times 4! \quad \left(= {}^{10}P_4 \times 3!\right)$ ***(A1)***

$= 30240$ ***A1***

[3 marks]

5. [Maximum mark: 7]

(a) [3]

Markscheme

recognition that $\left|\log_2 c\right| < 1$ ***(M1)***

$0.5 < c < 2, \quad (c \neq 1)$ ***A1A1***

Note: Award ***A1*** for endpoints and ***A1*** for strict inequalities.

[3 marks]

(b) [4]

Markscheme

attempt to find $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{5}{1-\log_2(1.5)} \quad \left(= \frac{5}{1-0.58496\ldots} = 12.0471\ldots\right)$ ***(A1)***

attempt to solve their $\left|S_\infty - S_n\right| < 0.1$ ***(M1)***

$\left|\frac{5}{1-\log_2(1.5)} - \sum_{r=0}^{n-1} 5\left(\log_2(1.5)\right)^r\right| < 0.1$ OR

$\left|\frac{5}{1-\log_2(1.5)} - \frac{5\left(1-\left(\log_2(1.5)\right)^n\right)}{1-\log_2(1.5)}\right| < 0.1$

Note: Award ***(M1)*** for solving an equality. Condone absence of absolute value signs.

$n = 8.93574\ldots$

$n = 9$ ***A1***

[4 marks]

6. [Maximum mark: 5]

(a) [3]

Markscheme

attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r + 5)$ ***(M1)***

$\tan 58^\circ = \frac{18.2}{r+5}$ OR $\frac{r+5}{\sin 32^\circ} = \frac{18.2}{\sin 58^\circ}$ OR $(r + 5 =)11.3726\ldots$ ***(A1)***

$r = 6.37262\ldots$ (m)

$(r =)\ 6.37$ (m) ***A1***

[3 marks]

(b) [2]

Markscheme

attempt to substitute $h = 20$ and their radius into the correct volume of cone formula ***(M1)***

$V = \frac{\pi(6.37262\ldots)^2(20)}{3}$

$= 850.540\ldots$

$= 851\,(\mathrm{m})^3$ ***(A1)***

Note: Accept $849.840\ldots$ (850) obtained from using $r = 6.37$.

[2 marks]

7. [Maximum mark: 6]

(a) [2]

Markscheme

recognition that velocity is zero ***(M1)***

$v = 2\sin(0.5t) + 0.3t - 2 = 0$

$t = 1.68694\ldots$

$t = 1.69$ ***A1***

[2 marks]

(b) [2]

Markscheme

$1.68694\ldots < t < 6.11857\ldots$

$1.69 < t < 6.12$ ***A1A1***

Note: Award ***A1*** for both values, ***A1*** for correct inequalities.

[2 marks]

(c) [2]

Markscheme

attempt to substitute into the total displacement formula (condone missing or incorrect limits, and absence of
$\mathrm{d}\ t$) ***(M1)***

$\int_0^{10}(2\ \sin\ (0.5t) + 0.3t - 2)\ \mathrm{d}\ t$ OR $\int_0^{10} v(t)\ \mathrm{d}\ t$

$= -2.13464\ldots$

$= -2.13\,(\mathrm{m})$ ***A1***

Note: Award ***(M1)A0*** if -2.13 is followed by 2.13.

[2 marks]

8. [Maximum mark: 8]

(a) [2]

Markscheme

(as $\lim\limits_{x\to 0} x^2 = 0$, the indeterminate form $\frac{0}{0}$ is required for the limit to exist)

$\Rightarrow \lim\limits_{x\to 0}(\arctan(\cos x) - k) = 0$ ***M1***

$\arctan 1 - k = 0\ \ (k = \arctan 1)$ ***A1***

so $k = \frac{\pi}{4}$ ***AG***

Note: Award ***M1A0*** for using $k = \frac{\pi}{4}$ to show the limit is $\frac{0}{0}$.

[2 marks]

(b) [6]

Markscheme

$$\lim_{x \to 0} \frac{\arctan(\cos x) - \frac{\pi}{4}}{x^2} \left(= \frac{0}{0}\right)$$

$$= \lim_{x \to 0} \frac{\frac{-\sin x}{1+\cos^2 x}}{2x}$$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

recognises to apply l'Hôpital's rule again ***(M1)***

$$= \lim_{x \to 0} \frac{\frac{-\sin x}{1+\cos^2 x}}{2x} \left(= \frac{0}{0}\right)$$

Note: Award ***M0*** if their limit is not the indeterminate form $\frac{0}{0}$.

EITHER

$$= \lim_{x \to 0} \frac{\frac{-\cos x\left(1+\cos^2 x\right) - 2\sin^2 x \cos x}{\left(1+\cos^2 x\right)^2}}{2}$$ ***A1A1***

Note: Award ***A1*** for a correct first term in the numerator and ***A1*** for a correct second term in the numerator.

OR

$$\lim_{x \to 0} \frac{-\cos x}{2(1+\cos^2 x) - 4x \sin x \cos x}$$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

THEN

substitutes $x=0$ into the correct expression to evaluate the limit ***A1***

Note: The final ***A1*** is dependent on all previous marks.

$=-\frac{1}{4}$ ***AG***

[6 marks]

9. [Maximum mark: 19]

(a.i) [1]

Markscheme

$$\int_{2.25}^{4.5} \frac{4}{21}\left(1-\cos\left(\frac{4\pi}{9}(t-2.25)\right)\right)\ \mathrm{d}\,t$$

$=\frac{3}{7}(=0.428571\ldots=0.479)$ ***A1***

[1 mark]

(a.ii) [1]

Markscheme

mode of $T=4.5$ ***A1***

[1 mark]

(a.iii) [2]

Markscheme

the median is greater ***A1***

$\mathrm{P}(T < 4.5) < 0.5$ **OR** $\mathrm{P}(T < 4.5) > 0.5$ **OR**
$\mathrm{P}(T < 4.5) < \mathrm{P}(T > 4.5)$ **OR** median $= 4.69$ ***R1***

Note: Accept reference to areas rather than probabilities.

[2 marks]

(b) [2]

Markscheme

recognition of the need to integrate f ***(M1)***

$$\int_{2.25}^{3.5} f(t)\ \mathrm{d}\ t$$

$$= 0.103749\ldots$$

$$= 0.104$$ ***A1***

[2 marks]

(c) [3]

Markscheme

attempt to use formula for conditional probability in context ***(M1)***

$\frac{\mathrm{P}(T \le 3)}{\mathrm{P}(\text{fast})}$ **OR** $\frac{\mathrm{P}(T \le 3)}{\mathrm{P}(T \le 3.5)}$ **OR** $\frac{\mathrm{P}(\text{very fast})}{\mathrm{P}(\text{fast})}$ (accept strict inequality signs)

$= \frac{0.0247152\ldots}{0.103749\ldots}$ ***(A1)***

$= 0.238220...$

$= 0.238$ ***A1***

[3 marks]

(d) [3]

Markscheme

recognition that the lower quartile q is the value such that

$\int_{2.25}^{q} f(t)\ \mathrm{d}\ t = 0.25$ ***(M1)***

$\int_{2.25}^{q} \frac{4}{21}\left(1 - \cos\left(\frac{4\pi}{9}(t - 2.25)\right)\right)\ \mathrm{d}\ t = 0.25$ ***(A1)***

Note: Condone the absence of $\mathrm{d}\ t$ for this ***A1***.

$\left[\frac{4}{21}\left(t - \frac{9}{4\pi}\sin\left(\frac{4\pi}{9}(t - 2.25)\right)\right)\right]_{2.25}^{q} = 0.25$

$q = 4.01290...$

$q = 4.01$ ***A1***

[3 marks]

(e) [5]

Markscheme

attempt to find the expected value for a transformed linear variable ***(M1)***

$\mathrm{E}(P) = \mathrm{E}(a - bT) = a - b\mathrm{E}(T)$

$a - 4.723b = 100$ *(A1)*

recognition that max score is achieved with fastest time $t = 2.25$ *(M1)*

maximum score $a - 2.25b = 150$ *(A1)*

$a = 195.491\ldots,\ b = 20.2183\ldots$

$a = 195,\ b = 20$ ***A1***

Note: these values must be given to the nearest integer for the ***A1*** to be awarded.

[5 marks]

(f) [2]

Markscheme

attempt to find variance for a transformed linear variable *(M1)*

$\text{Var }(P) = \text{Var }(a - bT) = b^2 \text{ Var }(T)$

$0.906\left(20.2183\ldots^2\right) = 370.356\ldots$

$\text{Var }(P) = 370$ ***A1***

Note: accept any answer which rounds to any value between 362 and 370 inclusive based on use of less accurate values of b.

[2 marks]

10. [Maximum mark: 17]

(a) [3]

Markscheme

METHOD 1

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\,\mathrm{AM}$

attempts to use Pythagoras' theorem to find AM^2 OR AM ***(M1)***

$\mathrm{AM}^2 = 20^2 - 14^2\ (= 204)$ OR
$\mathrm{AM} = \sqrt{20^2 - 14^2}\ \left(= 14.2828\ldots = \sqrt{204} = 2\sqrt{51}\right)$

recognizes that $\mathrm{AB} = 2\,\mathrm{AM}$ ***(A1)***

$\mathrm{AB} = 2 \times 14.2828\ldots\ (= 28.5657\ldots)\ \left(= 2\sqrt{204} = 4\sqrt{51}\right)$
A1

$\mathrm{AB} = 28.5657\ldots$

$\mathrm{AB} = 28.57$ (m) ***AG***

METHOD 2

let M be the midpoint of $[\mathrm{AB}]$ and so $\mathrm{AB} = 2\,\mathrm{AM}$

let $\theta = \mathrm{A\widehat{S}M}$

$\theta = 0.795398\ldots\ \left(= \cos^{-1}\frac{14}{20}\right)$ ***(A1)***

attempts to use a valid trigonometric ratio ***(M1)***

EITHER

$\mathrm{AM} = 14\tan\left(0.795398\ldots\right)\left(= 14.2828\ldots = 14\tan\left(\cos^{-1}\frac{14}{20}\right)\right)$
A1

OR

$\text{AM} = 20\sin(0.795398\ldots)\ \left(= 14.2828\ldots = 20\sin\left(\cos^{-1}\frac{14}{20}\right)\right)$ ***A1***

THEN

$\text{AB} = 28.5657\ldots$

$\text{AB} = 28.57$ (m) ***AG***

[3 marks]

(b) [1]

Markscheme

EITHER

the sprinkler rotates through (an angle of) 2π (radians) every 16 seconds and hence rotates through $\frac{2\pi}{16}$ (radians) in 1 second ***A1***

OR

$\left(\frac{2\pi}{n} = 16 \Rightarrow n =\right)\frac{2\pi}{16}\left(= \frac{\pi}{8}\right)$ ***A1***

THEN

sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second ***AG***

[1 mark]

(c) [4]

Markscheme

Note: For candidates that used Method 2 in part (a) apply full **FT** from their value of θ.

attempts to find 2θ where $\theta = \mathrm{A\hat{S}M}$ ***(M1)***

$= 2(0.795398\ldots)\ \left(1.59079\ldots = 2\cos^{-1}\frac{14}{20}\right)$

uses $\frac{\theta}{t}$ (rad/s) or similar to form an equation involving T ***(M1)***

$\frac{2\pi}{16} = \frac{1.59079\ldots}{T}\ \left(\frac{2\pi}{16} = \frac{2\cos^{-1}\frac{14}{20}}{T}\right)$ ***(A1)***

$T = 4.05093\ldots\ \left(= \frac{1.59079\ldots}{\frac{2\pi}{16}}\right)\left(= \frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}}\right)$

$T = 4.05$ (s) ***A1***

[4 marks]

(d) [1]

Markscheme

$\alpha = \frac{\pi t}{8}$ ***A1***

[1 mark]

(e) [3]

Markscheme

applies sine rule in $\Delta\mathrm{ASD}$ ***A1***

$\frac{d}{\sin\alpha} = \frac{20}{\sin \mathrm{A\hat{D}S}}$

attempts to find $\mathrm{A\hat{D}S}$ in terms of α ***M1***

$\mathrm{A\hat{D}S} = \pi - \beta - \alpha\ (= \pi - 0.7754 - \alpha)(= 2.366\ldots - \alpha)\ (= 2.37 - \alpha)$

$d = \frac{20\sin\alpha}{\sin(2.366\ldots - \alpha)}\ \left(= \frac{20\sin\alpha}{\sin(2.37 - \alpha)}\right)$ (accept $d = \frac{20\sin\alpha}{\sin(\pi - \beta - \alpha)}$) ***A1***

$$d = \frac{20 \sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 = \frac{\pi t}{8}\right)}$$ ***AG***

[3 marks]

(f) [1]

Markscheme

18 (m) ***A1***

[1 mark]

(g.i) [1]

Markscheme

$$w = \left|0.05t^2 + 1.1t + 18 - \frac{20 \sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}\right|$$ ***A1***

[1 mark]

(g.ii) [3]

Markscheme

attempts to solve $w = 0$ for t ***(M1)***

$t = 3.34880\ldots (12.7765\ldots)$

$t = 3.35$ (s) ***A1***

$22.2444\ldots$

22.2 (m) (south of A) ***A1***

[3 marks]

11. [Maximum mark: 21]

(a) [3]

Markscheme

attempts to use Euler's method ***(M1)***

$x_{n+1} = x_n + \frac{\pi}{12}$; $y_{n+1} = y_n + \frac{\pi}{12} \times \frac{\mathrm{d}y}{\mathrm{d}x}$ where $\frac{\mathrm{d}y}{\mathrm{d}x} = y \text{ cosec } 2x + \sqrt{\tan x}$

$y_1 = 1.25281\ldots \left(= \frac{\pi}{4} + \frac{\pi}{12}\left(\frac{\pi}{4} + 1\right)\right)$ ***(A1)***

$y_2 = 1.97608\ldots$

$y = 1.98$ ***A1***

[3 marks]

(b) [4]

Markscheme

attempts chain rule differentiation with multiplication of two derivatives ***(M1)***

$u = \cot x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\text{cosec}^2 x$ and $y = \frac{1}{2} \ln u \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}u} = \frac{1}{2u}$ OR

$u = \frac{\cos x}{\sin x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{\sin^2 x}$ and $y = \frac{1}{2} \ln u \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}u} = \frac{1}{2u}$ OR

$u = \sqrt{\cot (x)} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{\cot (x)}} \times -\text{cosec}^2 x$ and $\frac{\mathrm{d}y}{\mathrm{d}u} = \frac{1}{\sqrt{\cot (x)}}$ OR

$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2} \ln f(x)\right) = \frac{f\prime(x)}{2f(x)}$

THEN

$= -\frac{\text{cosec}^2 x}{2 \cot x}$ OR $-\frac{1}{\sin^2 x} \times \frac{\sin x}{2 \cos x}$ ***A1***

$= -\frac{1}{2 \sin x \cos x}$ ***(A1)***

$= -\frac{1}{\sin 2x}$ ***A1***

$- \operatorname{cosec} 2x$ ***AG***

[4 marks]

(c) [4]

Markscheme

METHOD 1

attempts to use $I(x) = \mathrm{e}^{\int P(x)\mathrm{d}x}$ ***(M1)***

$\mathrm{e}^{\int - \operatorname{cosec} 2x \mathrm{d}x}$ ***A1***

$= \mathrm{e}^{\frac{1}{2} \ln (\cot x)}$ ***(A1)***

$= \mathrm{e}^{\ln \left(\sqrt{\cot x}\right)}$ ***A1***

$= \sqrt{\cot x}$ ***AG***

METHOD 2

attempts product rule differentiation on $\frac{\mathrm{d}}{\mathrm{d}x}\left(y\sqrt{\cot x}\right)$ ***M1***

$= \frac{\mathrm{d}y}{\mathrm{d}x}\sqrt{\cot x} - \frac{y \operatorname{cosec}^2 x}{2\sqrt{\cot x}}$ ***A1***

$= \sqrt{\cot x}\left(\frac{\mathrm{d}y}{\mathrm{d}x} - y\frac{\operatorname{cosec}^2 x}{2 \cot x}\right)$ ***A1***

$= \sqrt{\cot x}\left(\frac{\mathrm{d}y}{\mathrm{d}x} - y \operatorname{cosec} 2x\right)$ ***A1***

so $\sqrt{\cot x}$ is an integrating factor ***AG***

[4 marks]

(d) [5]

Markscheme

$\sqrt{\cot x}\frac{\mathrm{d}y}{\mathrm{d}x} - y \text{ cosec } 2x\sqrt{\cot x} = \sqrt{\tan x}\sqrt{\cot x}$ (or equivalent) ***(M1)***

$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\sqrt{\cot x}\right) = 1$ ***(A1)***

$y\sqrt{\cot x} = \int 1 \text{ d } x$ ***A1***

$y\sqrt{\cot x} = x(+C)$ or equivalent ***A1***

substitutes $x = \frac{\pi}{4},\ y = \frac{\pi}{4} \Rightarrow C = 0$ ***M1***

Note: Award ***M1*** for attempting to find their value of C.

$y = x\sqrt{\tan x}$ ***AG***

[5 marks]

(e)

(e.i) [1]

Markscheme

$y = 2.52878\ldots$

$y = 2.53$ ***A1***

[1 mark]

(e.ii) [1]

Markscheme

the gradient changes substantially (in the neighbourhood of $x = \frac{5\pi}{12}$) ***R1***

Note: Award ***R0*** for saying the gradient is very large at $x = \frac{5\pi}{12}$

[1 mark]

(e.iii) [1]

Markscheme

EITHER

the curve is concave up (over the interval) ***A1***

OR

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ (over the interval) ***A1***

[1 mark]

(f) [2]

Markscheme

$\frac{\mathrm{d}y}{\mathrm{d}x} = y \operatorname{cosec} 2x + \sqrt{\tan x}\left(= x\sqrt{\tan x} \operatorname{cosec} 2x + \sqrt{\tan x}\right)$

$\operatorname{cosec} 2x,\ \sqrt{\tan x} > 0$ (for $0 < x < \frac{\pi}{2}$) ***R1***

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} > 0$ ***A1***

so the curve has a positive gradient for $0 < x < \frac{\pi}{2}$ ***AG***

Note: Do not award ***R0A1***.

[2 marks]