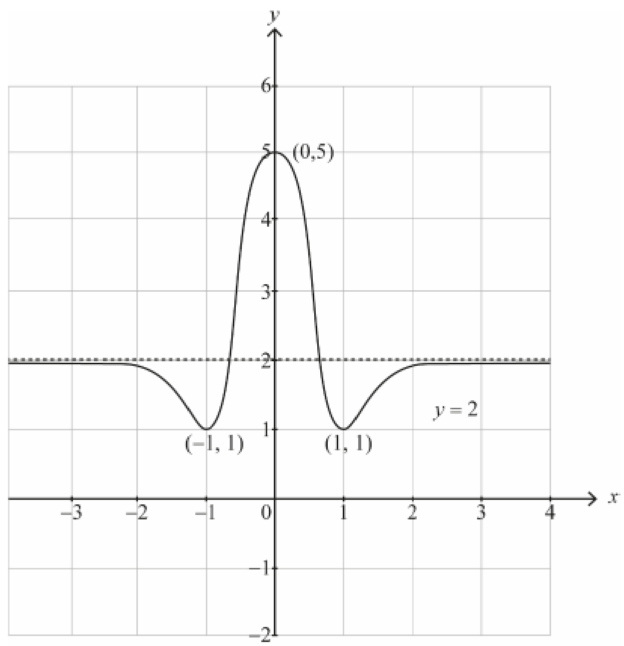
AAHL_Paper_1_QP [110 marks]

1. [Maximum mark: 5]

[5]

Markscheme



no y values below 1 ***A1***

horizontal asymptote at $y = 2$ with curve approaching from below as $x \to \pm\infty$ ***A1***

(±1,1) local minima ***A1***

(0,5) local maximum ***A1***

smooth curve and smooth stationary points ***A1***

[5 marks]

2. [Maximum mark: 7]

[7]

Markscheme

attempting integration by parts, *eg*

$u = \frac{\pi x}{36}, \mathrm{d}u = \frac{\pi}{36}\mathrm{d}x, \mathrm{d}v = \sin\left(\frac{\pi x}{6}\right)\mathrm{d}x, v = -\frac{6}{\pi}\cos\left(\frac{\pi x}{6}\right)$ ***(M1)***

P(0 ≤ *X* ≤ 3) $= \frac{\pi}{36}\left(\left[-\frac{6x}{\pi}\cos\left(\frac{\pi x}{6}\right)\right]_0^3 + \frac{6}{\pi}\int_0^3 \cos\left(\frac{\pi x}{6}\right)\mathrm{d}x\right)$ (or equivalent) ***A1A1***

Note: Award ***A1*** for a correct uv and ***A1*** for a correct $\int v\,\mathrm{d}u$.

attempting to substitute limits ***M1***

$\frac{\pi}{36}\left[-\frac{6x}{\pi}\cos\left(\frac{\pi x}{6}\right)\right]_0^3 = 0$ ***(A1)***

so P(0 ≤ *X* ≤ 3) $= \frac{1}{\pi}\left[\sin\left(\frac{\pi x}{6}\right)\right]_0^3$ (or equivalent) ***A1***

$= \frac{1}{\pi}$ ***A1***

[7 marks]

3. [Maximum mark: 7]

[7]

Markscheme

recognition that the angle between the normal and the line is 60° (seen anywhere) ***R1***

attempt to use the formula for the scalar product ***M1***

cos 60° $= \frac{\left|\begin{pmatrix}2\\1\\2\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\p\end{pmatrix}\right|}{\sqrt{9}\times\sqrt{1+4+p^2}}$ ***A1***

$\frac{1}{2} = \frac{|2p|}{3\sqrt{5+p^2}}$ ***A1***

$3\sqrt{5+p^2} = 4\,|p|$

attempt to square both sides ***M1***

$9\left(5+p^2\right) = 16p^2 \Rightarrow 7p^2 = 45$

$p = \pm 3\sqrt{\frac{5}{7}}$ (or equivalent) ***A1A1***

[7 marks]

4. [Maximum mark: 8]

(a) [3]

Markscheme

attempt to differentiate and set equal to zero ***M1***

$f'(x) = 2e^{2x} - 6e^x = 2e^x(e^x - 3) = 0$ ***A1***

minimum at $x = \ln 3$

$a = \ln 3$ ***A1***

[3 marks]

(b) [5]

Markscheme

Note: Interchanging x and y can be done at any stage.

$y = (e^x - 3)^2 - 4$ ***(M1)***

$e^x - 3 = \pm\sqrt{y+4}$ ***A1***

as $x \leqslant \ln 3, x = \ln\left(3 - \sqrt{y+4}\right)$ ***R1***

so $f^{-1}(x) = \ln\left(3 - \sqrt{x+4}\right)$ ***A1***

domain of f^{-1} is $x \in \mathbb{R}, -4 \leqslant x < 5$ ***A1***

[5 marks]

5. [Maximum mark: 5]

[5]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

METHOD 1

from vertex P, draws a line parallel to $[\mathrm{QR}]$ that meets $[\mathrm{SR}]$ at a point X **(M1)**

uses the sine rule in $\Delta\mathrm{PSX}$ **M1**

$\frac{\mathrm{PS}}{\sin\beta} = \frac{y-x}{\sin(180^\circ - \alpha - \beta)}$ **A1**

$\sin(180^\circ - \alpha - \beta) = \sin(\alpha + \beta)$ **(A1)**

$\mathrm{PS} = \frac{(y-x)\ \sin\ \beta}{\sin\ (\alpha+\beta)}$ **A1**

METHOD 2

let the height of quadrilateral PQRS be h

$h = \mathrm{PS}\ \sin\ \alpha$ **A1**

attempts to find a second expression for h **M1**

$h = (y - x - \mathrm{PS}\ \cos\ \alpha)\ \tan\ \beta$

$\mathrm{PS}\ \sin\ \alpha = (y - x - \mathrm{PS}\ \cos\ \alpha)\ \tan\ \beta$

writes $\tan\ \beta$ as $\frac{\sin\ \beta}{\cos\ \beta}$, multiplies through by $\cos\ \beta$ and expands the RHS **M1**

$\mathrm{PS}\ \sin\ \alpha\ \cos\ \beta = (y - x)\ \sin\ \beta - \mathrm{PS}\ \cos\ \alpha\ \sin\ \beta$

$\mathrm{PS} = \frac{(y-x)\ \sin\ \beta}{\sin\ \alpha\ \cos\ \beta+\cos\ \alpha\ \sin\ \beta}$ **A1**

$\mathrm{PS} = \frac{(y-x)\ \sin\ \beta}{\sin\ (\alpha+\beta)}$ **A1**

[5 marks]

6. [Maximum mark: 5]

(a) [3]

Markscheme

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attempts to calculate $\begin{pmatrix} 2 \\ 1 \\ m \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ -m \end{pmatrix}$ **(M1)**

$= -1 - m^2$ **A1**

since $m^2 \geq 0, \quad -1 - m^2 < 0$ for $m \in \mathbb{R}$ **R1**

so l_1 and l_2 are never perpendicular to each other **AG**

[3 marks]

(b) [2]

Markscheme

(since l_1 is parallel to Π, l_1 is perpendicular to the normal of Π and so)

$$\begin{pmatrix} 2 \\ 1 \\ m \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} = 0$$ **R1**

$2 + 4 - m = 0$

$m = 6$ **A1**

[2 marks]

7. [Maximum mark: 8]

(a) [3]

Markscheme

$x^2 + 6x + 10 = x^2 + 6x + 9 + 1 = (x+3)^2 + 1$ ***M1A1***

So the denominator is never zero and thus there are no vertical asymptotes. (or use of discriminant is negative) ***R1***

[3 marks]

(b) [2]

Markscheme

$x \to \pm\infty,\ f(x) \to 0$ so the equation of the horizontal asymptote is $y = 0$ ***M1A1***

[2 marks]

(c) [3]

Markscheme

$\int\limits_0^1 \frac{2x+6}{x^2+6x+10}\, dx = \left[\ln\left(x^2+6x+10\right)\right]_0^1 = \ln 17 - \ln 10 = \ln \frac{17}{10}$ ***M1A1A1***

[3 marks]

8. [Maximum mark: 9]

(a) [4]

Markscheme

$f'\left(x\right)=\frac{\left(2x-10\right)\left(x+1\right)-\left(x^2-10x+5\right)1}{\left(x+1\right)^2}$ ***M1***

$f'\left(x\right)=0\Rightarrow x^2+2x-15=0\Rightarrow\left(x+5\right)\left(x-3\right)=0$ ***M1***

Stationary points are $\left(-5,\ -20\right)$ and $\left(3,\ -4\right)$ ***A1A1***

[4 marks]

(b) [1]

Markscheme

$x=-1$ ***A1***

[1 mark]

(c) [4]

Markscheme

Looking at the nature table

x		-5		-1		3	
$f'(x)$	+ve	0	-ve	undefined	-ve	0	+ve

M1A1

$\left(-5,\ -20\right)$ is a max and $\left(3,\ -4\right)$ is a min ***A1A1***

[4 marks]

9. [Maximum mark: 21]

(a) [4]

Markscheme

$2t+1\times0+0\times\left(3+t\right)\ \left(=2t\right)$ (seen anywhere) ***(A1)*** one correct magnitude $\sqrt{1^2+1^2+0^2}$, $\sqrt{(2t)^2+(3+t)^2}$ ***(A1)***

correct substitution of their magnitudes and scalar product ***M1***

$2t=\sqrt{2}\times\sqrt{(2t)^2+(3+t)^2}\times\cos\frac{\pi}{3}$ OR $\cos\frac{\pi}{3}=\frac{2t}{\sqrt{2}\times\sqrt{5t^2+6t+9}}$

$4t=\sqrt{2(4t^2+9+6t+t^2)}$ OR $\frac{1}{2}=\frac{2t}{\sqrt{2(5t^2+6t+9)}}$ (or equivalent) ***A1***

$4t=\sqrt{10t^2+12t+18}$ ***AG***

[4 marks]

(b) [4]

Markscheme

correct quadratic equation ***A1***

$16t^2=10t^2+12t+18$, $6t^2-12t-18=0$, $t^2-2t-3=0$

valid attempt to solve their quadratic set $=0$ ***(M1)***

$(t+1)(t-3)$ OR $\frac{12\pm\sqrt{(-12)^2-4\times6\times(-18)}}{12}$ OR $(t-1)^2-4$ ***(A1)***

$t=3$ ***A1***

Note: Award ***A0*** if additional answer(s) given.

[4 marks]

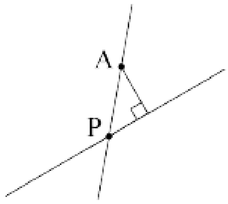
(c) [4]

Markscheme

METHOD 1

recognizing shortest distance from A is perpendicular to L_1 ***(M1)***

OR



$\sin\frac{\pi}{3} = \frac{x}{\left|\overrightarrow{\mathrm{PA}}\right|}$

$\left|\overrightarrow{\mathrm{PA}}\right| = \sqrt{6^2+6^2}$ $\left(= \sqrt{72},\ 6\sqrt{2}\right)$ (seen anywhere) ***(A1)***

$\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{72}}$ ***(A1)***

$x = \frac{\sqrt{216}}{2}$ $\left(= \sqrt{54},\ 3\sqrt{6}\right)$

shortest distance is $\frac{\sqrt{216}}{2}$ $\left(= \sqrt{54},\ 3\sqrt{6}\right)$ ***A1***

METHOD 2

recognition that the distance required is $\frac{\left|\boldsymbol{v}+\overrightarrow{\mathrm{PA}}\right|}{|\boldsymbol{v}|}$ ***(M1)***

$= \frac{1}{\sqrt{2}}\left|\begin{pmatrix}1\\1\\0\end{pmatrix} \times \begin{pmatrix}6\\0\\6\end{pmatrix}\right|$ ***(A1)***

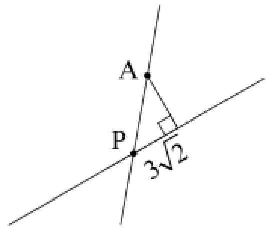
$= \frac{1}{\sqrt{2}}\left|\begin{pmatrix}6\\-6\\-6\end{pmatrix}\right|$ ***(A1)***

shortest distance is $\sqrt{54}$ $\left(= 3\sqrt{6}\right)$ ***A1***

METHOD 3

recognition that the base of the triangle is $\frac{\left|\boldsymbol{v}\cdot\overrightarrow{\mathrm{PA}}\right|}{|\boldsymbol{v}|}$ ***(M1)***

$= \frac{1}{\sqrt{2}}\left|\begin{pmatrix}1\\1\\0\end{pmatrix} \cdot \begin{pmatrix}6\\0\\6\end{pmatrix}\right|$

$= \frac{6}{\sqrt{2}} \left(= 3\sqrt{2}\right)$ OR  ***(A1)***

$\left|\overrightarrow{\mathrm{PA}}\right| = \sqrt{6^2 + 6^2} \quad \left(= \sqrt{72},\ 6\sqrt{2}\right)$ (seen anywhere) ***(A1)***

Note: The value of $\left|\overrightarrow{\mathrm{PA}}\right| = \sqrt{6^2 + 6^2}$ may be seen as part of the working of their shortest distance,

$d = \sqrt{\left|\overrightarrow{\mathrm{PA}}\right| - b^2} = \sqrt{\left(\sqrt{72}\right)^2 - \left(3\sqrt{2}\right)^2}$

shortest distance is $\sqrt{54} \left(= 3\sqrt{6}\right)$ ***A1***

METHOD 4

Let B be a general point on $L_1\ (\lambda,\ 8 + \lambda,\ \ -3)$ such that AB is perpendicular to L_1

attempt to find vector $\overrightarrow{\mathrm{AB}}$ OR $\left|\overrightarrow{\mathrm{AB}}\right|$ (the shortest distance from B to L_1) ***(M1)***

$$\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = \overrightarrow{\mathrm{OP}} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \overrightarrow{\mathrm{OA}} \left(= \begin{pmatrix} 0 \\ 8 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} = \overrightarrow{\mathrm{AP}} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right) \quad (\lambda \in \mathbb{R})$$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -6 \\ 0 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ OR $\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{(\lambda - 6)^2 + (8 + \lambda - 8)^2 + (-3 - 3)^2}$ ***A1***

$$\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{(\lambda - 6)^2 + \lambda^2 + (-6)^2} \quad \left(= \sqrt{2\lambda^2 - 12\lambda + 72}\right)$$

EITHER

$$\frac{d}{d\lambda}\left(\left|\overrightarrow{AB}\right|^2\right) = 0 \Rightarrow 4\lambda - 12 = 0 \Rightarrow \lambda = 3$$ ***A1***

OR

$\left|\overrightarrow{AB}\right| = \sqrt{2(\lambda - 3)^2 + 54}$ to obtain $\lambda = 3$ ***A1***

OR

$$\begin{pmatrix} -6+\lambda \\ \lambda \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0 \Rightarrow -6 + \lambda + \lambda = 0 \Rightarrow \lambda = 3$$ ***A1***

THEN

shortest distance is $\sqrt{54}\ \left(= 3\sqrt{6}\right)$ ***A1***

[4 marks]

(d) [2]

Markscheme

attempt to find the vector product of two direction vectors ***(M1)***

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix}$$

$\boldsymbol{n} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (or any scalar multiple of this) (accept $\boldsymbol{n} =< 1,\ -1,\ -1 >$ or equivalent) ***A1***

Note: Award ***A0*** for a final answer given in coordinate form.

[2 marks]

(e) [7]

Markscheme

substituting their x into volume formula and equating ***(M1)***

$\frac{1}{3}\pi\left(3\sqrt{6}\right)^2 h = 90\sqrt{3}\pi$

$h = 5\sqrt{3}$ (seen anywhere) ***A1***

recognition that the position vector of vertex is given by $\overrightarrow{OA} + \mu\boldsymbol{n}$ OR $\overrightarrow{\text{OA}} + h \times \hat{\boldsymbol{n}}$ ***(M1)***

$\begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ OR $(6+\mu,\ 8-\mu,\ 3-\mu)$

EITHER

recognition that $\mu|\boldsymbol{n}| = h$ (where μ is a parameter) ***(M1)***

$\mu|\boldsymbol{n}| = 5\sqrt{3}$ OR $\sqrt{\mu^2 + (-\mu)^2 + (-\mu)^2} = 5\sqrt{3}$ OR $3\mu^2 = 75$ $\left(\Rightarrow \sqrt{3}\mu = 5\sqrt{3}\right)$

$\mu = \pm 5$ (accept $\mu = 5$) ***(A1)***

OR

attempt to find cone's height vector $h \times \hat{\boldsymbol{n}}$ ***(M1)***

$\hat{\boldsymbol{n}} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ ***(A1)***

$5\sqrt{3} \times \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

THEN

$= \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} \pm 5\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \left(= \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} \pm \begin{pmatrix} 5 \\ -5 \\ -5 \end{pmatrix}\right)$

vertex = $(11, 3, -2)$ and $(1, 13, 8)$ (accept position vectors) ***A1A1***

Note: Award a maximum of ***(M0)A0(M1)(M1)(A1)A1A1FT*** for

$\left(\frac{39}{4}, \frac{17}{4}, -\frac{3}{4}\right)$ and $\left(\frac{9}{4}, \frac{47}{4}, \frac{27}{4}\right)$ obtained using $x = \left|\overrightarrow{\mathrm{PA}}\right|$ from part (c).

[7 marks]

10. [Maximum mark: 18]

(a.i) [2]

Markscheme

EITHER

attempt to use a ratio from consecutive terms ***M1***

$\frac{p \ln x}{\ln x} = \frac{\frac{1}{3}\ln x}{p \ln x}$ OR $\frac{1}{3}\ln x = (\ln x)r^2$ OR $p \ln x = \ln x\left(\frac{1}{3p}\right)$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \ldots$ and consider the powers of x in geometric sequence

Award ***M1*** for $\frac{p}{1} = \frac{\frac{1}{3}}{p}$.

OR

$r = p$ and $r^2 = \frac{1}{3}$ ***M1***

THEN

$p^2 = \frac{1}{3}$ OR $r = \pm\frac{1}{\sqrt{3}}$ ***A1***

$p = \pm\frac{1}{\sqrt{3}}$ ***AG***

Note: Award ***M0A0*** for $r^2 = \frac{1}{3}$ or $p^2 = \frac{1}{3}$ with no other working seen.

[2 marks]

(a.ii) [1]

Markscheme

EITHER

since, $|p| = \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}} < 1$ ***R1***

OR

since, $|p| = \frac{1}{\sqrt{3}}$ and $-1 < p < 1$ ***R1***

THEN

$\Rightarrow$ the geometric series converges. ***AG***

Note: Accept r instead of p.
Award ***R0*** if both values of p not considered.

[1 mark]

(a.iii) [3]

Markscheme

$\frac{\ln x}{1-\frac{1}{\sqrt{3}}} \quad \left(= 3 + \sqrt{3}\right)$ ***(A1)***

$\ln x = 3 - \frac{3}{\sqrt{3}} + \sqrt{3} - \frac{\sqrt{3}}{\sqrt{3}}$ OR $\ln x = 3 - \sqrt{3} + \sqrt{3} - 1 \ \ (\Rightarrow \ln x = 2)$ ***A1***

$x = \mathrm{e}^2$ ***A1***

[3 marks]

(b.i) [3]

Markscheme

METHOD 1

attempt to find a difference from consecutive terms or from u_2 ***M1***

correct equation ***A1***

$p \ln x - \ln x = \frac{1}{3}\ln x - p \ln x$ OR $\frac{1}{3}\ln x = \ln x + 2(p \ln x - \ln x)$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \ldots$ and consider the powers of x in arithmetic sequence.

Award ***M1A1*** for $p - 1 = \frac{1}{3} - p$

$2p \ln x = \frac{4}{3}\ln x \ \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 2

attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ ***M1***

$p \ln x = \frac{\ln x + \frac{1}{3}\ln x}{2}$ ***A1***

$2p \ln x = \frac{4}{3}\ln x \ \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 3

attempt to find difference using u_3 ***M1***

$\frac{1}{3}\ln x = \ln x + 2d \ \left(\Rightarrow d = -\frac{1}{3}\ln x\right)$

$u_2 = \ln x + \frac{1}{2}\left(\frac{1}{3}\ln x - \ln x\right)$ OR $p \ln x - \ln x = -\frac{1}{3}\ln x$ ***A1***

$p \ln x = \frac{2}{3}\ln x$ ***A1***

$p = \frac{2}{3}$ ***AG***

[3 marks]

(b.ii) [1]

Markscheme

$d = -\frac{1}{3}\ln x$ ***A1***

[1 mark]

(b.iii) [8]

Markscheme

METHOD 1

$S_n = \frac{n}{2}\left\lfloor 2\ \ln\ x + (n-1) \times \left(-\frac{1}{3}\ln\ x\right)\right\rfloor$

attempt to substitute into S_n and equate to $\ln\left(\frac{1}{x^3}\right)$ ***(M1)***

$\frac{n}{2}\left\lfloor 2\ \ln\ x + (n-1) \times \left(-\frac{1}{3}\ln\ x\right)\right\rfloor = \ln\left(\frac{1}{x^3}\right)$

$\ln\left(\frac{1}{x^3}\right) = -\ln\ x^3\left(= \ln\ x^{-3}\right)$ ***(A1)***

$= -3\ \ln\ x$ ***(A1)***

correct working with S_n (seen anywhere) ***(A1)***

$\frac{n}{2}\left\lfloor 2\ \ln\ x - \frac{n}{3}\ln\ x + \frac{1}{3}\ln\ x\right\rfloor$ OR $n\ \ln\ x - \frac{n(n-1)}{6}\ln\ x$ OR $\frac{n}{2}\left(\ln\ x + \left(\frac{4-n}{3}\right)\ln\ x\right)$

correct equation without $\ln\ x$ ***A1***

$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$ OR $n - \frac{n(n-1)}{6} = -3$ or equivalent

Note: Award as above if the series $1 + p + \frac{1}{3} + \ldots$ is considered leading to $\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$.

attempt to form a quadratic $= 0$ ***(M1)***

$n^2 - 7n - 18 = 0$

attempt to solve their quadratic ***(M1)***

$(n-9)(n+2) = 0$

$n = 9$ ***A1***

METHOD 2

$\ln\left(\frac{1}{x^3}\right) = -\ln\ x^3\left(= \ln\ x^{-3}\right)$ ***(A1)***

$= -3\ \ln\ x$ ***(A1)***

listing the first 7 terms of the sequence ***(A1)***

$\ln\ x + \frac{2}{3}\ln\ x + \frac{1}{3}\ln\ x + 0 - \frac{1}{3}\ln\ x - \frac{2}{3}\ln\ x - \ln\ x + \ldots$

recognizing first 7 terms sum to 0 ***M1***

8^{th} term is $-\frac{4}{3}\ln x$ ***(A1)***

9^{th} term is $-\frac{5}{3}\ln x$ ***(A1)***

sum of 8^{th} and 9^{th} term $= -3\ln x$ ***(A1)***

$n = 9$ ***A1***

[8 marks]

11. [Maximum mark: 17]

(a) [6]

Markscheme

let $t = \sqrt{x}$ ***M1***

$t^2 = x \Rightarrow 2t\,\mathrm{d}t = \mathrm{d}x$ ***A1***

so $\int \cos\sqrt{x}\mathrm{d}x = 2\int t\cos t\,\mathrm{d}t$ ***A1***

attempts integration by parts ***(M1)***

$u = 2t,\ \mathrm{d}\,v = \cos t\,\mathrm{d}t, \mathrm{d}u = 2\,\mathrm{d}t, v = \sin t$

$2\int t\cos t\,\mathrm{d}t = 2t\sin t - 2\int \sin t\,\mathrm{d}t$ ***(A1)***

$= 2t\sin t + 2\cos t + C$ ***A1***

substitution of $t = \sqrt{x} \Rightarrow \int \cos\sqrt{x}\mathrm{d}x = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$ ***AG***

[6 marks]

(b) [1]

Markscheme

$x_{n+1} = \frac{\left(2(n+1)-1\right)^2\pi^2}{4}\ \left(= \frac{(2n+1)^2\pi^2}{4}\right)$ ***A1***

[1 mark]

(c) [7]

Markscheme

area of R_n is $\left|\int_{x_n}^{x_{n+1}} \cos\sqrt{x}\,\mathrm{d}x\right|$ ***(M1)***

Note: Modulus may be seen at a later stage.

$= \left|\left[2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x}\right]_{\frac{(2n-1)^2\pi^2}{4}}^{\frac{(2n+1)^2\pi^2}{4}}\right|$ ***A1***

Note: Condone $+C$ at this stage.

attempts to substitute their limits into their integrated expression ***(M1)***

$= 2\left|\frac{(2n+1)\pi}{2} \times \sin\frac{(2n+1)\pi}{2} + \cos\frac{(2n+1)\pi}{2} - \left(\frac{(2n-1)\pi}{2} \times \sin\frac{(2n-1)\pi}{2} + \cos\frac{(2n-1)\pi}{2}\right)\right|$
A1

$= 2\left|(-1)^n\frac{(2n+1)\pi}{2} - \left((-1)^{n+1}\frac{(2n-1)\pi}{2}\right)\right|$ (or equivalent) ***A1***

$= 2\left|(-1)^n\frac{(2n+1)\pi}{2} + (-1)^n\frac{(2n-1)\pi}{2}\right|$ ***A1***

$= 2\left|(-1)^n\frac{4n\pi}{2}\right|$

$= 4n\pi$ ***A1***

Note: Award a maximum of ***(M1)A1M1A1A1A0A0*** for only attempting to calculate $\int_{x_n}^{x_{n+1}} \cos\sqrt{x}\,\mathrm{d}x$, and not applying the modulus.

[7 marks]

(d) [3]

Markscheme

EITHER

attempts to find $(d =) R_{n+1} - R_n$ ***M1***

$(d =) 4(n+1)\pi - 4n\pi$

$= 4\pi$ ***A1***

Note: Award ***M0*** for consideration of special cases for example R_3 and R_2. Accept $d = k\pi$.

which is a constant (common difference is 4π) ***R1***

OR

an arithmetic sequence is of the form $u_n = dn + c \ (u_n = dn + u_1 - d)$ ***M1***

attempts to compare $u_n = dn + c \ (u_n = dn + u_1 - d)$ and $R_n = 4n\pi$ ***M1***

$d = 4\pi$ and $c = 0 \ (u_1 - d = 0)$ ***A1***

Note: Accept $d = k\pi$.

THEN

so the areas of the regions form an arithmetic sequence ***AG***

[3 marks]